Thursday, February 8, 2018 2:31 PM

E.g. $\int \ln x \, dx$. $\begin{cases} u = \ln x \\ dv = dx \end{cases} \rightarrow \begin{cases} du = \frac{1}{x} dx \\ v = x \end{cases}$

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \ln x - \int dx = \boxed{x \ln x - x + C}$$

E.g. LIATE Rule does not work.

$\int x^3 e^{x^2} dx$. $\begin{cases} u = x^3 \\ dv = e^{x^2} dx \end{cases} \rightarrow \begin{cases} du = 3x^2 \\ \cancel{\int e^{x^2} dx = ?} \end{cases}$

Doesn't work

$\int x^2 \cdot x e^{x^2} dx$. $\begin{cases} u = x^2 \\ dv = x e^{x^2} dx \end{cases}$

$\rightarrow \begin{cases} du = 2x \, dx \\ v = \frac{1}{2} \int 2x e^{x^2} dx \end{cases}$

$w = x^2$. $dw = 2x \, dx$

$= \frac{1}{2} \int e^w dw = \frac{1}{2} e^w = \frac{1}{2} e^{x^2}$

Thursday, February 8, 2018 2:48 PM

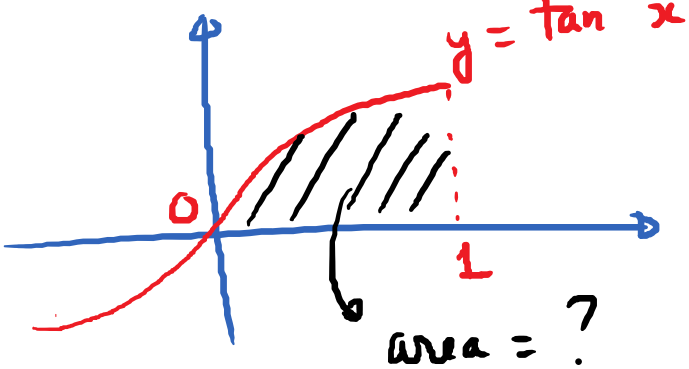
$$\int x^3 e^{x^2} dx = \frac{1}{2} x^2 e^{x^2} - \int x e^{x^2} dx$$

$$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C.$$

Integration By Parts For Definite Integrals

$$\boxed{\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du}$$

E.g. Find the area of the region bounded by the graph of $y = \tan^{-1} x$ and x-axis ; $0 \leq x \leq 1$



$$A = \int_0^1 \tan^{-1} x \, dx$$

Thursday, February 8, 2018 2:53 PM

$$\int_0^1 \tan^{-1} x \, dx$$

$$\begin{cases} u = \tan^{-1} x \\ dv = dx \end{cases} \rightarrow \begin{cases} du = \dfrac{1}{1+x^2}\,dx \\ v = x \end{cases}$$

$$\int_0^1 \tan^{-1} x \, dx = \underbrace{x \tan^{-1} x \Big|_0^1}_{} - \int_0^1 \frac{x}{1+x^2}\,dx$$

$$= 1 \cdot \tan^{-1}(1) - \cancel{0 \cdot \tan^{-1}(0)}\ 0$$

$$= \frac{\pi}{4}$$

$$\frac{1}{2}\int_0^1 \frac{2x}{1+x^2}\,dx \qquad \text{let } w = 1 + x^2. \quad dw = 2x\,dx$$

$$\frac{1}{2}\int_1^2 \frac{dw}{w} = \frac{1}{2} \cdot \ln|w| \Big|_1^2 = \frac{1}{2}\ln(2) - \frac{1}{2}\ln(1)$$

$$= \frac{1}{2}\ln(2)$$

Answer: $A = \boxed{\dfrac{\pi}{4} - \dfrac{1}{2}\ln(2)}$

Ex: Find the integral

1. $\int (x^3 + 2x) \cdot e^{2x} \, dx$

2. $\int \sin(\ln x) \, dx$

3. Find the volume of the solid of revolution: