$$\frac{5}{2}\arcsin\left(\frac{\sqrt{5}}{\sqrt{5}}\right) - \frac{5}{2}\arcsin\left(\frac{0}{\sqrt{5}}\right)$$

$$= \frac{5}{2}\arcsin(1) - \frac{5}{2}\arcsin(0)$$

$$= \frac{5}{2}\cdot\frac{\pi}{2} = \boxed{\frac{5\pi}{4}}$$

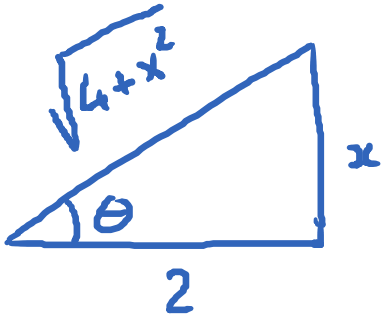
Eg. $\int \frac{dx}{\sqrt{4+x^2}}$ Trig Sub: $\boxed{x = 2\tan\theta}$

$dx = 2\sec^2\theta\, d\theta$

$$\int \frac{2\sec^2\theta}{\sqrt{4+4\tan^2\theta}}\, d\theta = \int \frac{2\sec^2\theta}{\sqrt{4(1+\tan^2\theta)}}\, d\theta$$

$$= \int \frac{2\sec^2\theta}{\sqrt{4\sec^2\theta}}\, d\theta = \int \frac{2\sec^2\theta}{2\sec\theta}\, d\theta$$

$$= \int \sec\theta\, d\theta = \ln\left|\sec\theta + \tan\theta\right| + C$$



$x = 2\tan\theta$

$\tan\theta = \frac{x}{2}$

$$\ln|\sec\theta + \tan\theta| + C = \boxed{\ln\left|\frac{\sqrt{4+x^2}}{2} + \frac{x}{2}\right| + C.}$$

E.g. HW #10.

$$\int \frac{dx}{(x^2-36)^{3/2}} = \int \frac{dx}{\left(\sqrt{x^2-36}\right)^3}$$

Trig sub: $\boxed{x = 6\sec\theta}$

$$dx = 6\sec\theta\tan\theta\, d\theta$$

$$\int \frac{6\sec\theta\tan\theta}{\left(\sqrt{36\sec^2\theta - 36}\right)^3} d\theta = \int \frac{6\sec\theta\tan\theta}{\left(\sqrt{36(\underbrace{\sec^2\theta - 1}_{\tan^2\theta})}\right)^3} d\theta$$

$$\int \frac{6\sec\theta\tan\theta}{\left(\sqrt{36\tan^2\theta}\right)^3}\,d\theta = \int \frac{6\sec\theta\tan\theta}{(6\tan\theta)^3}\,d\theta$$

$$= \int \frac{6\sec\theta\tan\theta}{216\tan^3\theta}\,d\theta = \frac{1}{36}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$= \frac{1}{36}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta = \frac{1}{36}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$$

$$= \frac{1}{36}\int \frac{\cos\theta\,d\theta}{\sin^2\theta}$$

$\cos\theta\,d\theta \to du$

$\sin^2\theta \to u^2$

$u = \sin\theta$

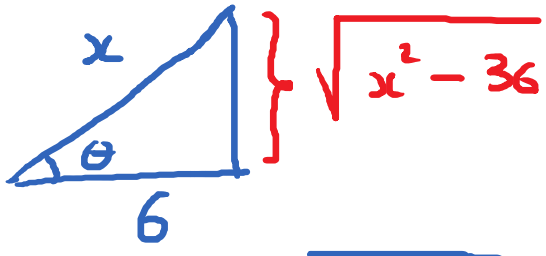
$du = \cos\theta\,d\theta$

$$= \frac{1}{36}\int \frac{du}{u^2} = \frac{1}{36}\int u^{-2}\,du = \frac{1}{36}\cdot\frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{36}\cdot\frac{1}{u} + C = -\frac{1}{36u} + C$$

$$= -\frac{1}{36\sin\theta} + C$$

Recall: $x = 6\sec\theta$



$$\sin\theta = \frac{\sqrt{x^2-36}}{x}$$

$$= -\frac{1}{36\cdot\frac{\sqrt{x^2-36}}{x}} + C$$

$$= \boxed{-\frac{x}{36\sqrt{x^2-36}} + C}$$

$x^2 + 4x + 4 = (x+2)^2$

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E.g. HW # 12.

$$\int \frac{dx}{\sqrt{x^2+4x-12}} = \int \frac{dx}{\sqrt{x^2+4x+4-16}}$$

$$= \int \frac{dx}{\sqrt{(x+2)^2-16}}$$ (Finish completing the square)

Trig sub: $x+2 = 4\sec\theta$

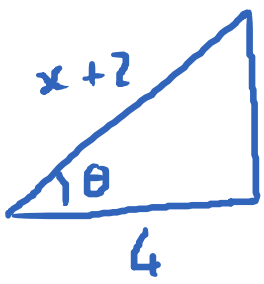
$dx = 4\sec\theta\tan\theta\, d\theta$

$$= \int \frac{4\sec\theta\tan\theta}{\sqrt{16\sec^2\theta - 16}}\, d\theta$$

$$= \int \frac{4\sec\theta\tan\theta}{\sqrt{16(\sec^2\theta - 1)}}\, d\theta = \int \frac{4\sec\theta\tan\theta}{4\tan\theta}\, d\theta$$

$$= \int \sec\theta \, d\theta = \ln\left|\sec\theta + \tan\theta\right| + C$$

$$x + 2 = 4\sec\theta \; ; \quad \sec\theta = \frac{x+2}{4}$$



$$\sqrt{(x+2)^2 - 16} \qquad \tan\theta = \frac{\sqrt{(x+2)^2 - 16}}{4}$$

$$\ln\left|\frac{x+2}{4} + \frac{\sqrt{(x+2)^2 - 16}}{4}\right| + C$$

$$= \boxed{\ln\left|\frac{x+2}{4} + \frac{\sqrt{x^2 + 4x - 12}}{4}\right| + C}$$