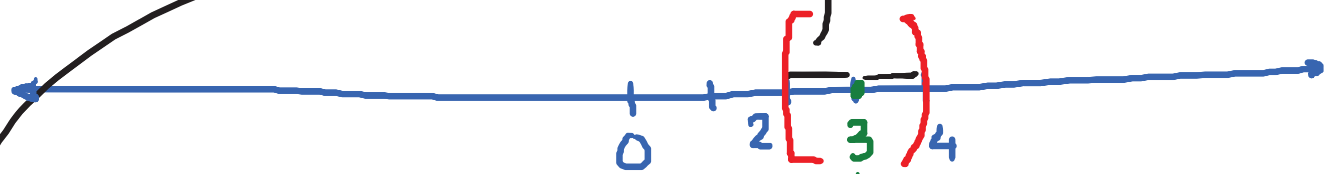


center of series

Distance from center to either endpoint = 1

The number 1 is called the $\boxed{\text{Radius of Convergence}}$ of the series. $R = 1$.

<u>Ex.</u> Consider the series

$$\sum_{n=1}^{\infty} \frac{(x-6)^n}{n^n}.$$

(Center of this series = 6)

<u>Q:</u> Values of x for which the series converges?

(Hint: Root Test)

Tuesday, April 3, 2018 2:33 PM

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left|\frac{(x-6)^n}{n^n}\right|} = \left|\frac{x-6}{n}\right|$$

$$= \frac{|x-6|}{n}$$

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{|x-6|}{n} = 0$$, regardless of what x is.

Since $0 < 1$, the series converges for all values of x.

<u>Conclusion:</u> Interval of convergence $(-\infty, \infty)$

Radius of convergence $R = \infty$

Theorem Given any power series $\sum_{n=0}^{\infty} c_n (x-a)^n$

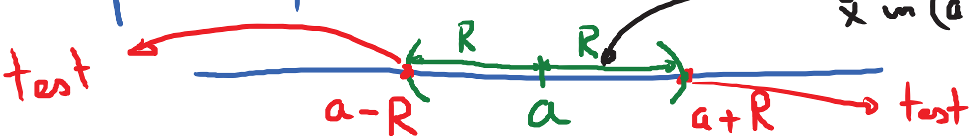
Exactly one of the following scenarios will happen:

1. The series converges only at the center $x = a$. For all $x \neq a$, the series diverges $(R=0)$

2. The series converges for all values of x. $(R = \infty)$

3. The series converges for every x within an interval surrounding the center. (It may converge or it may diverge at the endpoints of that interval)

converges for every x in $(a-R, a+R)$



* Represent Functions with Power Series

E.g. $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$

Series — Function

for $-1 < x < 1$.

In general

$$\boxed{\sum_{n=0}^{\infty} (\text{Stuff})^n = \frac{1}{1 - \text{Stuff}}}$$

provided $-1 < \text{Stuff} < 1$.

E.g. Use a power series to represent the given function. Find the interval of convergence.

① $f(x) = \frac{1}{1+x^3}$ ② $g(x) = \frac{x^2}{4-x^2}$.

(1) $f(x) = \dfrac{1}{1+x^3} = \dfrac{1}{1-(-x^3)}$

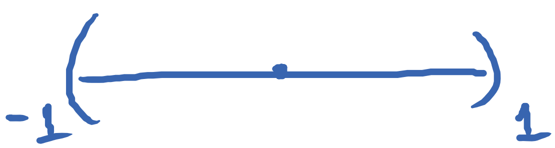
$$\frac{1}{1-(-x^3)} = \sum_{n=0}^{\infty} (-x^3)^n = \sum_{n=0}^{\infty} (-1)^n (x^3)^n$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot x^{3n}}$$

Interval of convergence: $-1 < -x^3 < 1$.

$$1 > x^3 > -1$$

$$1 > x > -1$$



Interval of convergence is $(-1, 1)$

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(2) $g(x) = \dfrac{x^2}{4-x^2} \cdot \dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \dfrac{1}{\frac{4}{x^2} - 1} \cdot \dfrac{(-1)}{(-1)}$

$$= \frac{-1}{1 - \frac{4}{x^2}} = -1 \cdot \frac{1}{1 - \frac{4}{x^2}}$$

$$-1\cdot\sum_{n=0}^{\infty}\left(\frac{4}{x^2}\right)^n = -\sum_{n=0}^{\infty}\frac{4^n}{x^{2n}}$$

$$= -\sum_{n=0}^{\infty} 4^n\cdot x^{-2n}.$$

$$g(x) = \frac{x^2}{4-x^2} = \frac{x^2}{4\left(1-\frac{x^2}{4}\right)} = \frac{x^2}{4}\cdot\frac{1}{1-\frac{x^2}{4}}$$

$$= \frac{x^2}{4}\cdot\sum_{n=0}^{\infty}\left(\frac{x^2}{4}\right)^n$$

$$= \frac{x^2}{4}\sum_{n=0}^{\infty}\frac{x^{2n}}{4^n}$$

this is a series which represent $g(x)$

$$= \sum_{n=0}^{\infty}\frac{x^2}{4}\cdot\frac{x^{2n}}{4^n} = \sum_{n=0}^{\infty}\frac{x^{2n+2}}{4^{n+1}}$$

Interval of convergence: $\left|\frac{x^2}{4}\right| < 1 \leftrightarrow |x^2| < 4$

$\leftrightarrow |x| < 2. \leftrightarrow$ I.O.C: $(-2,2)$

Ex. Find the interval of convergence and radius of convergence of the given series:

(1) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (2) $\sum_{n=0}^{\infty} \frac{(-3)^n \cdot x^n}{\sqrt{n+1}}$

Ex. Find the power series that represent the given function. Find interval of convergence.

(1) $f(x) = \frac{x^3}{2-x}$ (2) $g(x) = \frac{x^2}{1-4x^2}$.