Polar Coordinates and Calculus of Polar Curves

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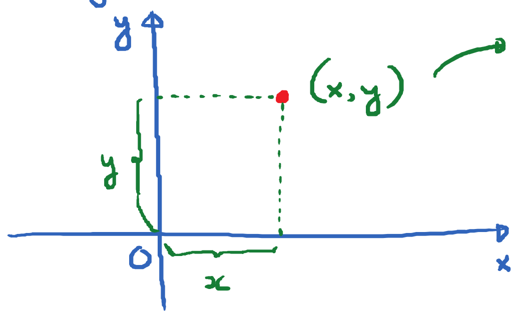
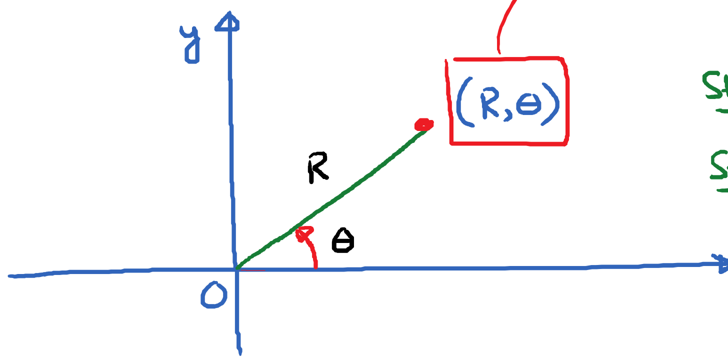
Rectangular Coordinates



→ rectangular coordinates of the point

Polar Coordinates



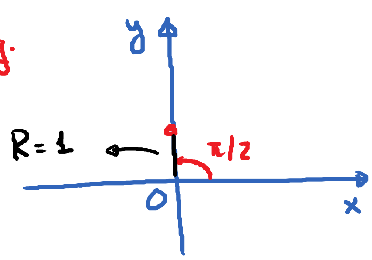
this pair of numbers is called the polar coordinates of the point

Step 1: Draw line segment from point to origin

Step 2: length of line segment = R

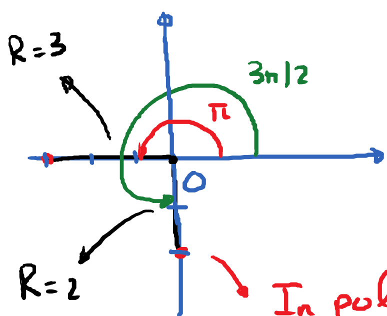
Step 3: Measure the angle from the positive part of x-axis to the ray connecting O to point. → θ

E.g.



In Rectangular coordinates: $(0, 1)$

In Polar coordinates: $\left(1, \frac{\pi}{2}\right)$



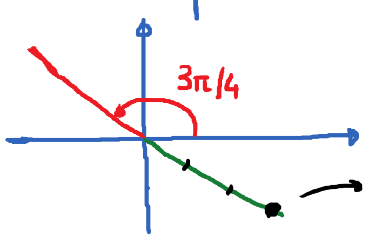
In Rectangular coordinates: $(-3, 0)$

In Polar coordinates: $(3, \pi)$

In polar coordinates: $\left(2, \frac{3\pi}{2}\right)$

Convention: when R is negative.

E.g. locate the point whose polar coordinates is $\left(-3, \frac{3\pi}{4}\right)$



→ $\left(-3, \frac{3\pi}{4}\right)$

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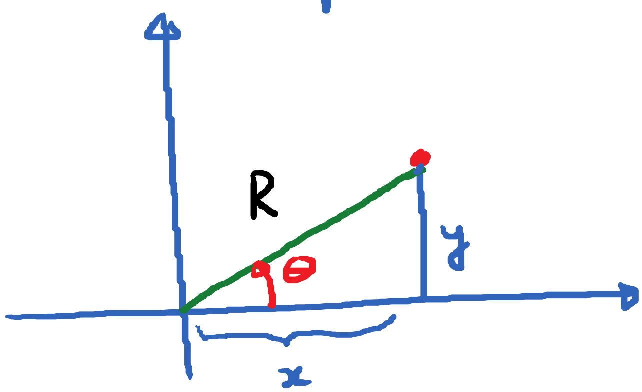
Relationship between Rectangular and Polar Coordinates



Polar → Rectangular

$$\frac{x}{R} = \frac{\text{adj.}}{\text{hyp.}} = \cos\theta$$

$$\rightarrow \boxed{x = R\cos\theta}$$

$$\frac{y}{R} = \frac{\text{opp.}}{\text{hyp.}} = \sin\theta$$

$$\rightarrow \boxed{y = R\sin\theta}$$

Polar → Rectangular: $\boxed{x = R\cos\theta \ ; \ y = R\sin\theta}$

Rectangular → Polar: $\boxed{R = \sqrt{x^2 + y^2}}$

$$\tan\theta = \frac{y}{x} \rightarrow \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

Note: $\tan^{-1}$ only gives angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Add π if necessary for 2^{nd} & 3^{rd} quadrant points.

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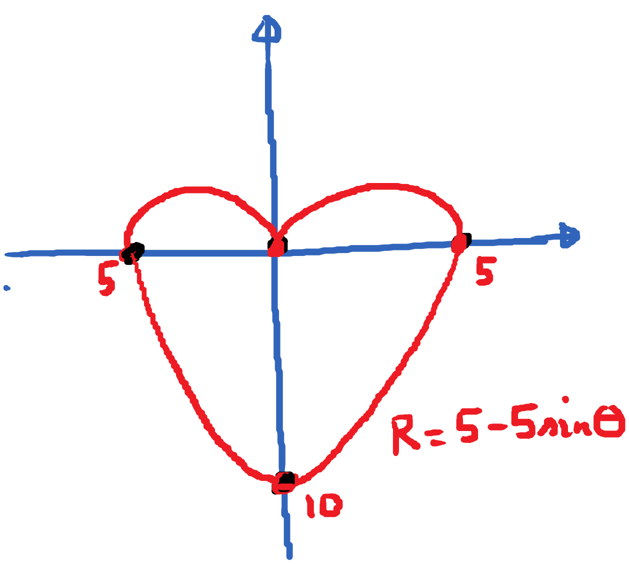
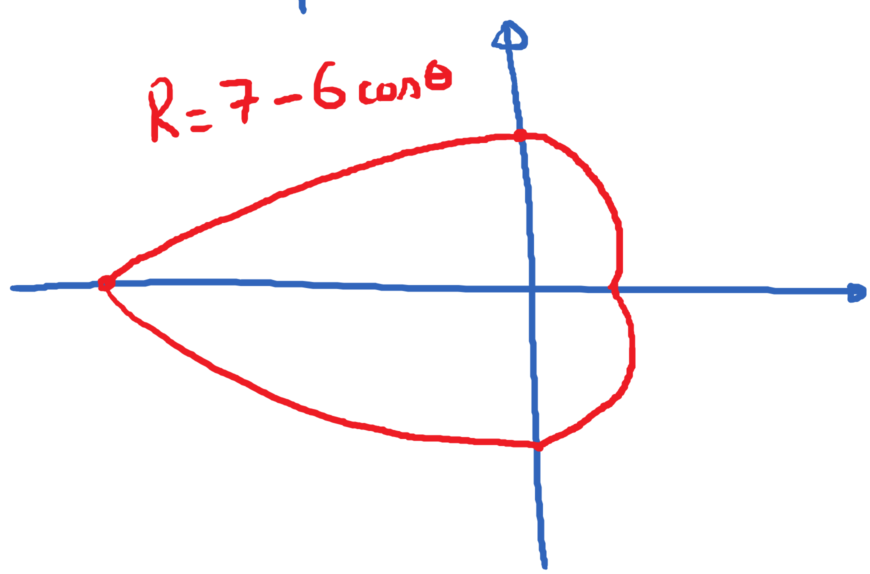
Graphs of equations in polar coordinates

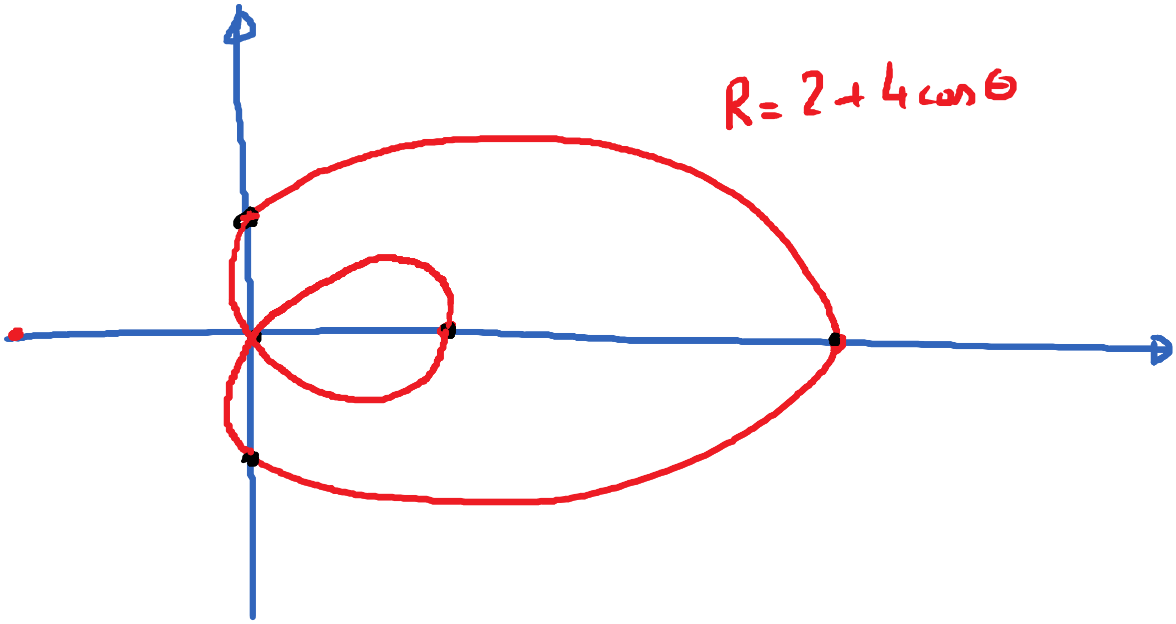
E.g. (1) $R = 5 - 5\sin\theta$ (2) $R = 7 - 6\cos\theta$

(3) $R = 2 + 4\cos\theta$

θ	$R = 5 - 5\sin\theta$	$R = 7 - 6\cos\theta$	$R = 2 + 4\cos\theta$
0	5	1	6
$\frac{\pi}{2}$	0	7	2
π	5	13	-2
$\frac{3\pi}{2}$	10	7	2
2π	5	1	6







Tangents with Polar Curves

Given a polar curve : $\boxed{R = f(\theta)}$

Slope of tangent line at a point $= \dfrac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\boxed{dy/d\theta}}{\boxed{dx/d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cdot\cos\theta}{f'(\theta)\cdot\cos\theta - f(\theta)\cdot\sin\theta}$$

$$y = R\sin\theta = f(\theta)\cdot\sin\theta \longrightarrow \frac{dy}{d\theta} = ?$$

$$x = R\cos\theta = f(\theta)\cdot\cos\theta \longrightarrow \frac{dx}{d\theta} = ?$$

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Product Rule

$$y = f(\theta)\cdot\sin\theta \rightarrow \frac{dy}{d\theta} = f'(\theta)\cdot\sin\theta + f(\theta)\cdot\cos\theta$$

$$x = f(\theta)\cdot\cos\theta \rightarrow \frac{dx}{d\theta} = f'(\theta)\cdot\cos\theta - f(\theta)\cdot\sin\theta$$

Formula for Slope of tangent line:

$$\boxed{\frac{dy}{dx} = \frac{f'(\theta)\cdot\sin\theta + f(\theta)\cdot\cos\theta}{f'(\theta)\cdot\cos\theta - f(\theta)\cdot\sin\theta}}$$

Recall: $R = f(\theta)$; $f'(\theta) = \frac{dR}{d\theta}$.

$$\boxed{\frac{dy}{dx} = \frac{\frac{dR}{d\theta}\cdot\sin\theta + R\cdot\cos\theta}{\frac{dR}{d\theta}\cdot\cos\theta - R\cdot\sin\theta}}$$