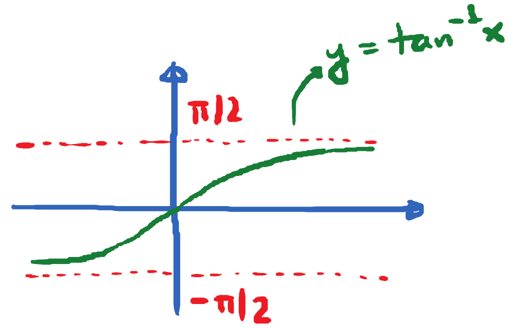
(7) $$\int_0^\infty \frac{4\left(\overbrace{\boxed{1+\tan^{-1}x}}^{u}\right)}{1+x^2}\,dx, \qquad \frac{dx}{1+x^2} \to du$$

Let $u = 1 + \tan^{-1}x \longrightarrow du = \dfrac{1}{1+x^2}\,dx$

$$\int 4u\,du = \frac{4u^2}{2} = 2u^2$$

$$\longrightarrow 2\left(1+\tan^{-1}x\right)^2 \Big|_0^\infty$$



$$\longrightarrow 2\left[\left(1+\underbrace{\tan^{-1}(\infty)}_{\frac{\pi}{2}}\right)^2 - \left(1+\tan^{-1}(0)\right)^2\right]$$

$$2\left[\left(1+\frac{\pi}{2}\right)^2 - 1\right] = 2\cdot\left[\cancel{1} + \pi + \frac{\pi^2}{4} - \cancel{1}\right]$$

$$= \boxed{2\pi\left(1+\frac{\pi}{4}\right)}$$

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$$\int \frac{\boxed{e^t\,dt}^{\,du}}{e^{2t}-7e^t+6}$$

Let $u = e^t$. $du = e^t\,dt$

$$\int \frac{du}{u^2-7u+6} = \int \frac{du}{(u-1)(u-6)}$$

Partial Fractions Decomposition:

$$\frac{1}{(u-1)(u-6)} = \frac{A}{u-1} + \frac{B}{u-6}$$

$$1 = A(u-6) + B(u-1)$$

Plug in $u = 6 \rightarrow 1 = 5B \rightarrow B = 1/5$

$u = 1 \rightarrow 1 = -5A \rightarrow A = -1/5$

$$\int \frac{du}{(u-1)(u-6)} = -\frac{1}{5}\int \frac{du}{u-1} + \frac{1}{5}\int \frac{du}{u-6}$$

$$= -\frac{1}{5}\ln|u-1| + \frac{1}{5}\ln|u-6|$$

$$= \boxed{-\frac{1}{5}\ln|e^t-1| + \frac{1}{5}\ln|e^t-6|} + C$$

(9) Maclaurin Series for e^{-7x}

$$e^{u} = \sum_{n=0}^{\infty} \frac{u^n}{n!}$$

$$\longrightarrow e^{-7x} = \sum_{n=0}^{\infty} \frac{(-7x)^n}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 7^n x^n}{n!}}$$

(10) Given: $\cot(x) = \frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \cdots\right)$

Find first 4 terms for the series for $\ln(\sin x)$

$$\ln(\sin x) = \int \cot(x)\,dx = \int \frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \cdots\right)dx$$

$$= \boxed{\ln|x| - \left(\frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{3 \cdot 945} + \cdots\right)}$$

(11) $x = 3\sin^3 t$; $y = 3\cos^3 t$; $0 \le t \le \pi$

$$\text{Length} = \int_0^\pi \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$x'(t) = 3 \cdot 3 \cdot \sin^2 t \cdot \cos t = 9\sin^2 t \cos t$$

$$y'(t) = 3 \cdot 3 \cdot \cos^2 t \cdot (-\sin t) = -9\cos^2 t \sin t$$

$$L = \int_0^\pi \sqrt{[9\sin^2 t \cos t]^2 + [-9\cos^2 t \sin t]^2}\, dt$$

$$= \int_0^\pi \sqrt{81\sin^4 t \cos^2 t + 81\cos^4 t \sin^2 t}\, dt$$

$$= \int_0^\pi \sqrt{81\sin^2 t \cos^2 t \underbrace{(\sin^2 t + \cos^2 t)}_{1}}\, dt$$

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$$= \boxed{\int_0^{\pi} 9 \sin t \cos t \, dt}$$

$$\boxed{\frac{d}{dx}(\sqrt{u}) = \frac{u'}{2\sqrt{u}}}$$

$$\left[(t+3)^{1/2}\right]' = \frac{1}{2}(t+3)^{-1/2}$$

$$= \frac{1}{2\sqrt{t+3}}$$

12 $x = \sqrt{t+3}$; $y = -t$

$$\frac{d^2y}{dx^2} = ? \quad \text{when } t = 13$$

1^{st} Step: $$\frac{dy}{dx} = \frac{dy/dt}{\boxed{dx/dt}} = \frac{-1}{\boxed{\frac{1}{2\sqrt{t+3}}}} = -2\sqrt{t+3}$$

2^{nd} Step: $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\boxed{\frac{dy}{dx}}\right)}{\frac{dx}{dt}} = \frac{\boxed{\frac{d}{dt}\left(-2\sqrt{t+3}\right)}}{\frac{1}{2\sqrt{t+3}}}$$

$$= \frac{-2 \cdot \cancel{\frac{1}{2\sqrt{t+3}}}}{\cancel{\frac{1}{2\sqrt{t+3}}}} = \boxed{-2}$$

Remember: To find $\frac{d^2y}{dx^2}$ of parametric equations.

① $\boxed{\frac{dy}{dx}} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$

② $\frac{\frac{d}{dt}\left(\boxed{\frac{dy}{dx}}\right)}{\frac{dx}{dt}} = \frac{d^2y}{dx^2}$.

13 $\int \overset{A}{\boxed{x^4}} \cdot \overset{L}{\boxed{\ln(4x)}}\, dx$ L.I.A.T.E

$\begin{cases} u = \ln(4x) \\ dv = x^4 dx \end{cases} \longrightarrow \begin{cases} du = \frac{1}{x} dx \\ v = \frac{x^5}{5} \end{cases}$.

Integration by parts: $\boxed{\int u\,dv = uv - \int v\,du}$

$\int x^4 \cdot \ln(4x) dx = \underbrace{\boxed{\frac{x^5}{5}}}_{v} \cdot \underbrace{\boxed{\ln(4x)}}_{u} - \int \underbrace{\boxed{\frac{x^5}{5}}}_{v} \cdot \underbrace{\boxed{\frac{1}{x} dx}}_{du}$

$$= \frac{x^5 \cdot \ln(4x)}{5} - \frac{1}{5}\int x^4\,dx$$

$$= \frac{x^5 \cdot \ln(4x)}{5} - \frac{1}{5} \cdot \frac{x^5}{5} + C$$

$$= \boxed{\frac{x^5 \cdot \ln(4x)}{5} - \frac{x^5}{25} + C}$$

14 $$\int \frac{8x^2 + x + 63}{x^3 + 9x}\,dx = \int \frac{8x^2 + x + 63}{x(x^2+9)}\,dx$$

Form of Partial Fractions Decomposition:

$$\boxed{\frac{8x^2 + x + 63}{x(x^2+9)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 9}}$$

15 $\displaystyle\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{4^n}$. Find I.O.C.

Root Test: $a_n = \dfrac{\left[(x-4)^2\right]^n}{4^n}$

$$\sqrt[n]{|a_n|} = \sqrt[n]{\left|\frac{\left[(x-4)^2\right]^n}{4^n}\right|} = \frac{(x-4)^2}{4}$$

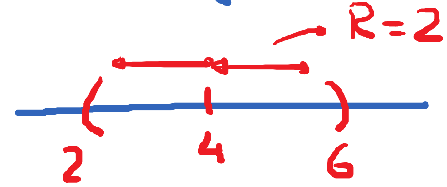
$$\lim_{n\to\infty} \frac{(x-4)^2}{4} = \frac{(x-4)^2}{4} < 1$$

$\longrightarrow (x-4)^2 < 4$

$\longrightarrow -2 < x-4 < 2$

$\longrightarrow 2 < x < 6$

$$\boxed{\text{I.O.C: } (2,6)}$$



Check endpoints: $x=6$: $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{4^n}{4^n} = \sum_{n=0} 1$,

$x=2$: $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{4^n}{4^n} = \sum_{n=0}^{\infty} 1 \nearrow$ Diverges.