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$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{1}{n}\right)}{n^2}$$

$$f(x) = \frac{\cos\left(\frac{1}{x}\right)}{x^2}$$ on $[1, \infty)$

positive ✓

decreasing ✓

cont. ✓

$$-\int_1^{\infty} \frac{\cos\left(\boxed{\frac{1}{x}}^{u}\right)}{\boxed{-x^2}}\boxed{dx} \to du$$

let $u = \frac{1}{x}$; $du = -\frac{1}{x^2}dx$

$$-\int \cos(u)\,du = -\sin(u) = -\sin\left(\frac{1}{x}\right)\Bigg|_1^{\infty}$$

$$= \underbrace{-\sin\left(\boxed{\frac{1}{\infty}} \to 0\right)}_{0} + \sin(1)$$

$= \sin(1)$ → Integral converges

→ $\boxed{\text{Series converges.}}$

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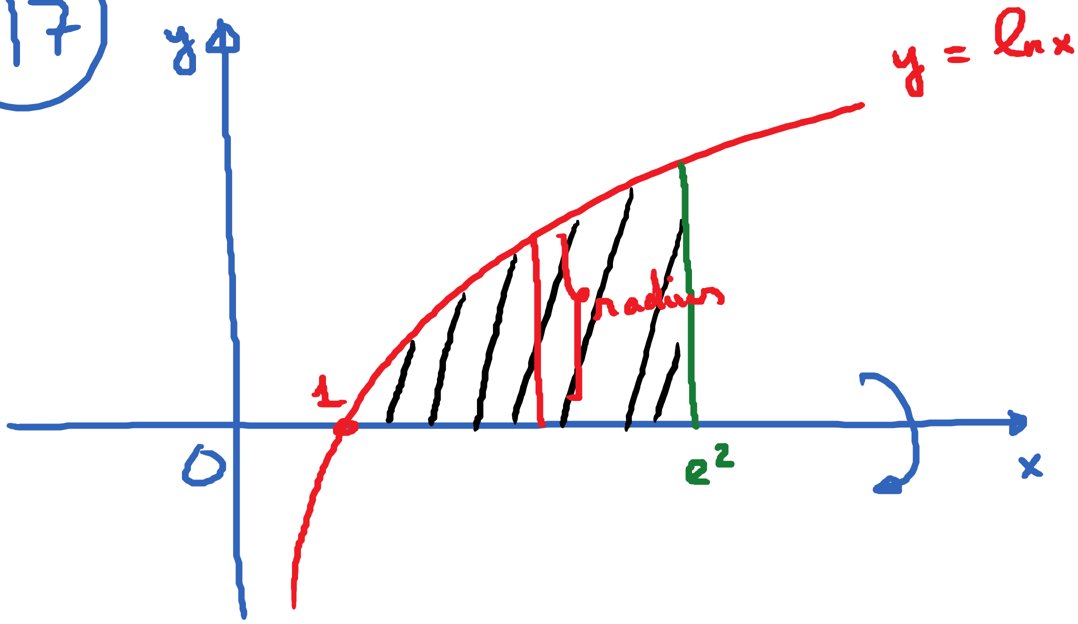

$$V_{solid} = \int \pi \cdot (\text{radius})^2$$

$$V = \int_1^{e^2} \pi \cdot (\ln x)^2\, dx = \pi \cdot \int_1^{e^2} (\ln x)^2\, dx$$

⟶ Integration by parts

$$\begin{cases} u = (\ln x)^2 \\ dv = dx \end{cases} \longleftrightarrow \begin{cases} du = 2 \cdot \ln x \cdot \frac{1}{x}\, dx \\ v = x \end{cases}$$

$$\pi \cdot \int_1^{e^2} (\ln x)^2\,dx = \pi \cdot \left(\underbrace{\boxed{x}}_{v} \underbrace{\boxed{(\ln x)^2}}_{u} \Bigg|_1^{e^2} - \int_1^{e^2} \underbrace{\boxed{\cancel{x}}}_{v} \cdot \underbrace{\boxed{2 \cdot \ln x \cdot \frac{1}{\cancel{x}}\,dx}}_{du} \right)$$

$$= \pi \cdot \left(e^2 \cdot (\ln(e^2))^2 - 2 \cdot \int_1^{e^2} \ln x\,dx \right)$$

(**Note:** $\boxed{\int \ln x = x \ln x - x}$

Why? $\int \ln x\,dx$ $\left\{ \begin{array}{l} u = \ln x \\ dv = dx \end{array} \right. \longleftrightarrow \left\{ \begin{array}{l} du = \frac{1}{x}\,dx \\ v = x \end{array} \right.$

$$\int \ln x\,dx = x \ln x - \int x \cdot \frac{1}{x}\,dx$$

$$= x \ln x - \int dx = x \ln x - x \quad)$$

$$\pi \cdot \left(e^2 \cdot (2)^2 - 2 \cdot \left[x \ln x - x \right] \Big|_1^{e^2} \right)$$

$$= \pi \cdot \left(4e^2 - 2 \cdot \left[e^2 \cdot \ln(e^2) - e^2 - (-1) \right] \right)$$

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$$= \pi \cdot \left(4e^2 - 2\left[2e^2 - e^2 + 1\right] \right)$$

$$= \pi \cdot \left(4e^2 - 2e^2 - 2 \right) = \pi\left(2e^2 - 2\right) = \boxed{2\pi\left(e^2 - 1\right)}$$

(18) $R = 6\sqrt{1+\cos(2\theta)}\ ; \ 0 \leq \theta \leq \frac{\pi}{4}$.

$$L = \int_0^{\pi/4} \sqrt{R^2 + \left(\frac{dR}{d\theta}\right)^2}\, d\theta$$

$$\frac{dR}{d\theta} = 6 \cdot \frac{\left(-2\sin(2\theta)\right)}{2\sqrt{1+\cos(2\theta)}} = \frac{-6\sin(2\theta)}{\sqrt{1+\cos(2\theta)}}.$$

$$L = \int_0^{\pi/4} \sqrt{36\left(1+\cos(2\theta)\right) + \frac{36\sin^2(2\theta)}{1+\cos(2\theta)}}\, d\theta$$

$$L = 6\int_0^{\pi/4} \sqrt{\frac{\left[1+\cos(2\theta)\right]^2 + \sin^2(2\theta)}{1+\cos(2\theta)}}\, d\theta$$

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$$L = 6 \cdot \int_0^{\pi/4} \sqrt{\frac{2 + 2\cos(2\theta)}{1 + \cos(2\theta)}}\, d\theta = \boxed{6 \cdot \int_0^{\pi/4} \sqrt{2}\, d\theta.}$$

19 $$\tan^{-1}(u) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{u^{2n+1}}{2n+1}$$

So, $$\tan^{-1}(x^2) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{(x^2)^{2n+1}}{2n+1}$$

$$\frac{\tan^{-1}(x^2)}{x^2} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{4n+2}}{x^2(2n+1)}$$

$$= \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{4n}}{2n+1}.}$$