Final Review

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1. $x = t + \cos t$; $y = 2 - \sin t$.

Find equation of tangent line to curve when $t = \frac{\pi}{6}$

Point: $x = \frac{\pi}{6} + \cos\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \frac{\sqrt{3}}{2}$.

$y = 2 - \sin\left(\frac{\pi}{6}\right) = 2 - \frac{1}{2} = \frac{3}{2}$

$\longrightarrow$ Point $\left(\frac{\pi}{6} + \frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

Slope: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} = \frac{-\cos(t)}{1 - \sin(t)}$

Slope when $t = \frac{\pi}{6}$, $\frac{-\cos(\pi/6)}{1 - \sin(\pi/6)} = \frac{-\sqrt{3}/2}{1 - \frac{1}{2}}$

$= \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

Point-Slope Equation: $y - \frac{3}{2} = -\sqrt{3}\left(x - \left(\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right)\right)$

Simplify: $y = -\sqrt{3}x + \frac{\pi\sqrt{3}}{6} + \frac{3}{2} + \frac{3}{2}$

$$\boxed{y = -\sqrt{3}x + \frac{\pi\sqrt{3}}{6} + 3}$$

2 Polar curve: $R = e^{4\theta}$; $0 \leq \theta \leq \pi$

$$\text{Length} = \int_0^\pi \sqrt{R^2 + \left(\frac{dR}{d\theta}\right)^2}\, d\theta$$

$$\frac{dR}{d\theta} = 4e^{4\theta}$$

$$\longrightarrow \text{Length} = \int_0^\pi \sqrt{(e^{4\theta})^2 + (4e^{4\theta})^2}\, d\theta$$

$$= \int_0^\pi \sqrt{e^{8\theta} + 16e^{8\theta}}\, d\theta$$

$$= \int_0^\pi \sqrt{17e^{8\theta}}\, d\theta = \sqrt{17}\int_0^\pi e^{4\theta}\, d\theta$$

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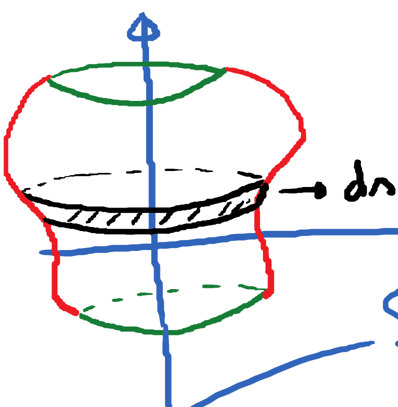
$$\longrightarrow \sqrt{17}\cdot\frac{e^{4\theta}}{4}\Bigg|_0^{\pi} = \boxed{\frac{\sqrt{17}}{4}\left(e^{4\pi}-1\right)}$$

③ $x = t+\sqrt{6}\ ;\ y = \frac{t^2}{2}+\sqrt{6}\,t\ ;\ -\sqrt{6}\le t\le\sqrt{6}.$

Revolve about y-axis



$$\text{Surface area} = \int 2\pi x\, ds$$

$$\text{Surface area} = 2\pi\int_{-\sqrt{6}}^{\sqrt{6}} x(t)\cdot\sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$$

$$x'(t) = 1\ ;\ y'(t) = t+\sqrt{6}$$

$$2\pi\cdot\int_{-\sqrt{6}}^{\sqrt{6}}(t+\sqrt{6})\cdot\sqrt{1+(t+\sqrt{6})^2}\,dt$$

$$= 2\pi\cdot\int_{-\sqrt{6}}^{\sqrt{6}}(t+\sqrt{6})\cdot\sqrt{1+t^2+2t\sqrt{6}+6}\,dt$$

$$= \boxed{2}\pi \int_{-\sqrt{6}}^{\sqrt{6}} \boxed{(t+\sqrt{6})} \cdot \sqrt{\boxed{t^2 + 2t\sqrt{6} + 7}} \, \boxed{dt}$$

$\longrightarrow u$

Let $u = t^2 + 2t\sqrt{6} + 7$. $du = (2t + 2\sqrt{6})\,dt$

$$\boxed{du = 2(t+\sqrt{6})\,dt}$$

$$\pi \int_1^{25} \sqrt{u}\, du = \pi \int_1^{25} u^{1/2}\, du$$

Change the bounds: $t = -\sqrt{6}$: $u = (-\sqrt{6})^2 + 2\cdot(-\sqrt{6})\cdot\sqrt{6} + 7$

$u = 6 - 12 + 7 = 1$

$t = \sqrt{6}$: $u = (\sqrt{6})^2 + 2(\sqrt{6})(\sqrt{6}) + 7$

$u = 6 + 12 + 7 = 25$

$$\pi \cdot \frac{2u^{3/2}}{3}\Bigg|_1^{25} = \frac{2\pi}{3}\left((25)^{3/2} - (1)^{3/2}\right)$$

$$= \frac{2\pi}{3}(125 - 1) = \boxed{\frac{248\pi}{3}}$$

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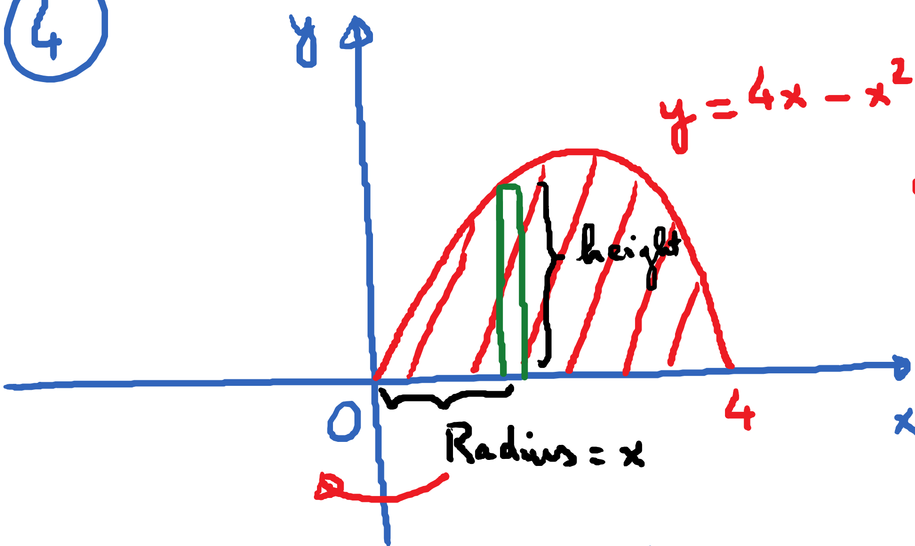

Shell method

$$V = 2\pi \int_a^b (\text{radius})(\text{height})$$

$$V_{solid} = 2\pi \int_0^4 x \cdot (4x - x^2)\, dx$$

$$= 2\pi \cdot \int_0^4 (4x^2 - x^3)\, dx$$

$$= 2\pi \cdot \left(\frac{4x^3}{3} - \frac{x^4}{4} \right) \Big|_0^4$$

$$= 2\pi \cdot \left(\frac{4}{3} \cdot (4)^3 - \frac{(4)^4}{4} \right)$$

$$= \boxed{\frac{128\pi}{3}}$$

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(5) $xy = 5$; $3 \le y \le 4$; Rotate about y-axis.

$$\text{Surface Area} = \int 2\pi x \, ds$$

$$x = \boxed{\frac{5}{y}}$$

$$ds \begin{cases} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \\ \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \end{cases}$$

$$\frac{dx}{dy} = -\frac{5}{y^2}$$

$$\text{Surface area} = 2\pi \int_3^4 \frac{5}{y} \cdot \sqrt{1 + \left(-\frac{5}{y^2}\right)^2}\, dy$$

$$= 2\pi \cdot \int_3^4 \frac{5}{y} \cdot \sqrt{1 + \frac{25}{y^4}}\, dy$$

$$= \boxed{10\pi \cdot \int_3^4 \frac{1}{y} \cdot \sqrt{1 + \frac{25}{y^4}}\, dy}$$

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$$L = \int \sqrt{1 + \left[\frac{dx}{dy}\right]^2}\, dy$$

$$\rightarrow \left[\frac{dx}{dy}\right]^2 = \frac{16}{y^3} \rightarrow \frac{dx}{dy} = \frac{4}{y^{3/2}} = 4 \cdot y^{-3/2}$$

$$\rightarrow dx = 4 \cdot y^{-3/2}\, dy$$

$$\rightarrow x = \int 4 \cdot y^{-3/2}\, dy = 4 \cdot \frac{y^{-1/2}}{-1/2} = -8y^{-1/2} + C$$

$$\rightarrow x = -\frac{8}{\sqrt{y}} + C$$

$(-8, 1)$ belongs to curve $\rightarrow x = -8 \,;\, y = 1$

$$-8 = -\frac{8}{\sqrt{1}} + C \rightarrow -8 = -8 + C$$

$$\rightarrow C = 0$$

$$\rightarrow \boxed{x = -\frac{8}{\sqrt{y}}}$$