(2) $|\text{Error}| \leq |5^{\text{th}} \text{ term}|$

5^{th} term $\longrightarrow$ plug in $n = 5 \longrightarrow \dfrac{(0.1)^{11}}{11} \approx 9.09 \cdot (10)^{-13}$

(3) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln n}{3n+7}\right)^n$.

<u>Root test:</u> $\sqrt[n]{|a_n|} = \sqrt[n]{\left|\dfrac{\ln n}{3n+7}\right|^n} = \left|\dfrac{\ln n}{3n+7}\right|$

$\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left|\frac{\ln n}{3n+7}\right|$ $\frac{\infty}{\infty}$ limit $\to$ L'Hopital

$\displaystyle = \lim_{n\to\infty} \left|\frac{\frac{1}{n}}{3}\right| = \lim_{n\to\infty} \left(\frac{1}{3n}\right) = 0 < 1$

$\longrightarrow$ converges.

$\displaystyle\lim_{n\to\infty} \sqrt[n]{|a_n|}$
- $0 \leq x < 1$: converges
- > 1 : diverges
- $= 1$: test fails.

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④ $$\sum_{n=0}^{\infty} \frac{n(n+1)(n+2)}{6^n} (x-\pi)^n .$$

Find R.O.C.

→ Ratio Test: $$\frac{a_{n+1}}{a_n} = \frac{\cancel{(n+1)}\cancel{(n+2)}(n+3)(x-\pi)^{\cancel{n}+1} \cdot \cancel{6^n}}{6^{n+1}\, n\cancel{(n+1)}\cancel{(n+2)}\cancel{(x-\pi)^n}}$$

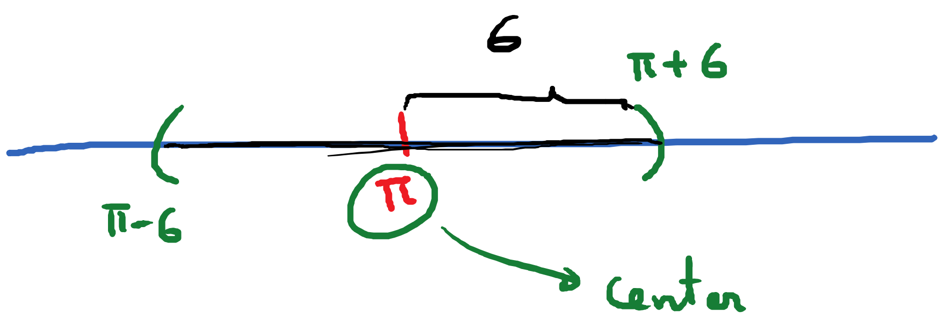
$$\frac{a_{n+1}}{a_n} = \frac{(n+3)(x-\pi)}{6n}$$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{n+3}{6n} |x-\pi| = \frac{|x-\pi|}{6} < 1$$

(For series to converge)

→ $|x-\pi| < 6$

Radius of Convergence is $\boxed{6}$.



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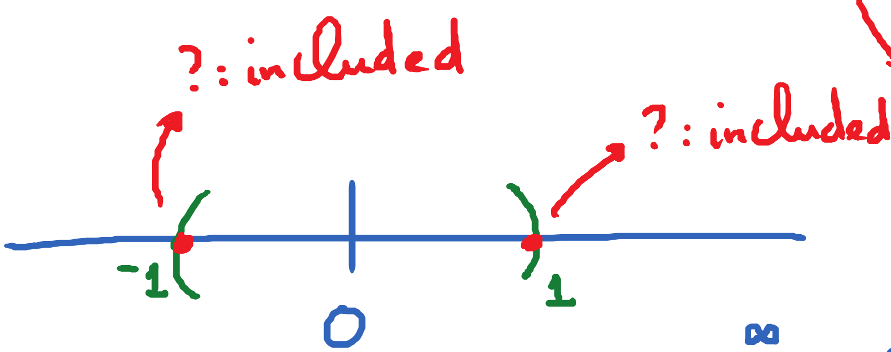
⑤ $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n^3+6}}$

Ratio Test: $\displaystyle\frac{a_{n+1}}{a_n} = \frac{x^{\cancel{n}+1}}{\sqrt{(n+1)^3+6}} \cdot \frac{\sqrt{n^3+6}}{\cancel{x^n}}$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \underbrace{\frac{\sqrt{n^3+6}}{\sqrt{(n+1)^3+6}}}_{\to 1} \cdot |x| = |x| < 1$$



Check $x=-1$. Series $= \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^3+1}} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{\sqrt{n^3+1}}$

$\longrightarrow$ converges by the AST

check $x = 1$. Series $= \displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$ $\rightarrow$ behaves like $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

p-series; $p = \frac{3}{2}$ $\longrightarrow$ converges

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(6) $$\underbrace{\sum_{n=0}^{\infty}\left(\frac{x^2+4}{3}\right)^n}_{1+\frac{x^2+4}{3}+\left(\frac{x^2+4}{3}\right)^2+\left(\frac{x^2+4}{3}\right)^3+\cdots} \longrightarrow \sum_{n=0}^{\infty}(\text{Stuff})^n = \frac{1}{1-(\text{Stuff})}$$

common ratio: $\frac{x^2+4}{3}$.

$$= \frac{1}{1-\frac{x^2+4}{3}} = \frac{1}{\frac{3-x^2-4}{3}} = \frac{3}{-1-x^2}$$

$$= -\frac{3}{1+x^2}$$

---

(7) Maclaurin series for $f(x) = e^{-7x}$.

$$\boxed{e^u = \sum_{n=0}^{\infty}\frac{u^n}{n!}}$$

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$$e^{-7x} = \sum_{n=0}^{\infty} \frac{(-7x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-7)^n \cdot x^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 7^n \cdot x^n}{n!}.$$

---

8 $f(x) = x^8 \cdot \sin x$. Taylor series centered at 0.

$$f(x) = x^8 \cdot \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{\boxed{x^8 \cdot x^{2n+1}}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+9}}{(2n+1)!}$$

9 Given $\cot x = \frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \dots\right)$

Q: Find the first 4 terms for the series of

$\ln(\sin x)$?

$$\frac{d}{dx}\left[\ln(\sin x)\right] = \frac{\cos x}{\sin x} = \cot x$$

$$\longrightarrow \int \cot x \, dx = \ln(\sin x)$$

$$\int \left[\frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \dots\right)\right] dx$$

$$= \ln|x| - \left(\frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \dots\right)$$

10 $\sin x = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$$\sin\left(\frac{2}{3}\right) = \frac{2}{3} - \frac{2^3}{3^3 \cdot 3!} + \frac{2^5}{3^5 \cdot 5!} - \frac{2^7}{3^7 \cdot 7!} + \dots$$