11

$$\sum_{n=1}^{\infty} (-1)^n \cdot \underbrace{\boxed{\ln\left(\frac{6n+3}{6n+2}\right)}}_{}$$

b_n

Decreasing

$\lim_{n\to\infty} b_n \overset{?}{=} 0$

$$\lim_{n\to\infty} \ln\left(\boxed{\frac{6n+3}{6n+2}}\right) = \ln(1) = 0$$

$\to 1$

$$\left[\ln\left(\frac{6n+3}{6n+2}\right)\right]' = \frac{\frac{6\cdot(6n+2) - 6\cdot(6n+3)}{(6n+2)^2}}{\frac{6n+3}{6n+2}}$$

$$= \frac{-6/(6n+2)^2}{\frac{6n+3}{6n+2}} < 0$$

$\longrightarrow$ decreasing.

By AST, converges.

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12 $\boxed{\dfrac{1}{1-x}} = \sum_{n=0}^{\infty} x^n \longrightarrow \dfrac{d}{dx}(x^n) = n x^{n-1}$

$\to$ Differentiate both sides of this equation w.r.t. x

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{d}{dx}\left(\left(\boxed{1-x}\right)^{-1}\right) = -1 \cdot (1-x)^{-2} \cdot (-1)$$

$$= \frac{1}{(1-x)^2}$$

$$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n \cdot x^{n-1}$$

Replace x by $\frac{1}{7}$.

$$\frac{1}{\left(1-\frac{1}{7}\right)^2} = \sum_{n=1}^{\infty} n \cdot \left(\frac{1}{7}\right)^{n-1}$$

$$\frac{49}{36} = \sum_{n=1}^{\infty} \frac{n}{7^{n-1}}$$

13 $\dfrac{\sin(5x)}{x}$

$$\sin(u) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{u^{2n+1}}{(2n+1)!}$$

So, $$\sin(5x) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{(5x)^{2n+1}}{(2n+1)!}$$

So, $$\frac{\sin(5x)}{x} = \frac{1}{\cancel{x}} \cdot \sum_{n=0}^{\infty} (-1)^n \cdot \frac{5^{2n+1} \cdot x^{2n\cancel{+1}}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{5^{2n+1} \cdot x^{2n}}{(2n+1)!}$$

14 $$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n \cdot x^n$$

$$\ln(1+x) = \int \frac{1}{1+x} dx = \sum_{n=0}^{\infty} (-1)^n \int x^n dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{n+1}}{n+1}.$$

So, $\ln(1+x^2) = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{(x^2)^{n+1}}{n+1}$

So, $\frac{\ln(1+x^2)}{x} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{n+1}$.

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(16) $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{((x-4)^2)^n}{4^n}$

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(x-4)^2}{4}\right|^n}$

$= \lim_{n\to\infty} \left|\frac{(x-4)^2}{4}\right| = \frac{(x-4)^2}{4}$

Want: $\frac{(x-4)^2}{4} < 1 \rightarrow \frac{|x-4|}{2} < 1$

$|x-4| < 2 \rightarrow -2 < x-4 < 2$

$\rightarrow \boxed{2} < x < \boxed{6}$. Check endpoints.

Plug in $x=2$: $\sum_{n=0}^{\infty} \frac{(-2)^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{((-2)^2)^n}{4^n}$

$= \sum_{n=0}^{\infty} \frac{4^n}{4^n} = \sum_{n=0}^{\infty} 1 \to$ diverges.

Plug in $x=6$ $\sum_{n=0}^{\infty} \frac{2^{2n}}{4^n} = \sum_{n=0}^{\infty} \frac{4^n}{4^n} = \sum_{n=0}^{\infty} 1 \to$ diverges

I.O.C: $(2,6)$

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(17) Want: $|(n+1) \text{ term}| < 0.001$

$$\frac{1}{(4(n+1)+1)^3} < \frac{1}{1000}$$

$$\frac{1}{(4n+5)^3} < \frac{1}{1000}.$$

$$(4n+5)^3 > 1000$$

$$4n+5 > 10 \to 4n > 5$$

$\to n > \frac{5}{4} = 1.25 \to$ enough to use: $n=2$ terms.

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$$\frac{x / x^2}{(1+x^2)/x^2} = \frac{1}{x} \cdot \frac{1}{1+\frac{1}{x^2}}$$

$$= \frac{1}{x} \cdot \frac{1}{1-\left(-\frac{1}{x^2}\right)}$$

$$= \frac{1}{x} \cdot \sum_{n=0}^{\infty} \left(-\frac{1}{x^2}\right)^n = \frac{1}{x} \cdot \sum_{n=0}^{\infty} (-1)^n \cdot x^{-2n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot x^{-2n-1}$$

$$\frac{1}{2}\int \frac{2x}{1+x^2}\,dx \qquad u = 1+x^2 \,;\; du = 2x\,dx$$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(1+x^2)$$

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2)$$

$$\ln(1+x^2) = 2 \cdot \int \frac{x}{1+x^2}\,dx$$

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$$\ln(1+x^2) = 2 \cdot \sum_{n=0}^{\infty} (-1)^n \cdot \int x^{-2n-1}\,dx$$

$$= \cancel{2} \cdot \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{-2n}}{-\cancel{2}n}$$

$$= -\sum_{n=0}^{\infty} \frac{(-1)^n}{n x^{2n}}.$$