2414 - Practice 4 (Final) - Spr18

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question. Write your choice in the space provided. No partial credit.

Find an equation for the line tangent to the curve at the point defined by the given value of t.

1) $x = t + \cos t,\ y = 2 - \sin t,\ t = \frac{\pi}{6}$ 1) _____

A) $y = -\sqrt{3}x + \frac{3}{2}$

B) $y = -\sqrt{3}x + \frac{\sqrt{3}}{6}\pi + 3$

C) $y = -\sqrt{2}x - \frac{\sqrt{2}}{4}\pi + 2$

D) $y = \sqrt{3}x - \frac{\sqrt{3}}{6}\pi$

Find the length of the curve.

2) The spiral $r = e^{4\theta},\ 0 \le \theta \le \pi$ 2) _____

A) $\frac{1}{16}(e^{8\pi} - 1)$ B) $\frac{\sqrt{17}}{4}(e^{4\pi} - 1)$ C) $\frac{32}{3}(\pi^2 - 1)$ D) $e^{8\pi} - \sqrt{3}$

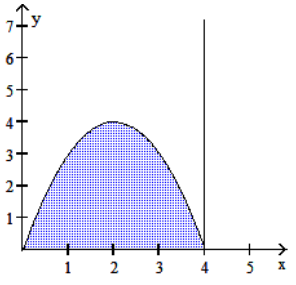
Find the area of the surface generated by revolving the curves about the indicated axis.

3) $x = t + \sqrt{6},\ y = \frac{t^2}{2} + \sqrt{6}t,\ -\sqrt{6} \le t \le \sqrt{6};$ y-axis 3) _____

A) $\frac{248}{3}\pi$ B) 124π C) $\frac{124}{3}\pi$ D) $\frac{248}{5}\pi$

Use the shell method to find the volume of the solid generated by revolving the shaded region about the indicated axis.

4) About the y-axis 4) _____



$y = 4x - x^2$

A) $\frac{128}{3}\pi$ B) 32π C) 64π D) $\frac{64}{3}\pi$

Set up an integral for the area of the surface generated by revolving the given curve about the indicated axis.

5) $xy = 5,\ 3 \le y \le 4;$ y-axis 5) _____

A) $5\pi \int_3^4 \frac{1}{y}\sqrt{1 + 25y^{-4}}\,dy$

B) $5\pi \int_3^4 \frac{1}{y}\sqrt{1 + 5y^{-4}}\,dy$

C) $10\pi \int_3^4 \frac{1}{y}\sqrt{1 + 5y^{-4}}\,dy$

D) $10\pi \int_3^4 \frac{1}{y}\sqrt{1 + 25y^{-4}}\,dy$

Solve the problem.

6) Find a curve through the point (-8, 1) whose length integral, $1 \le y \le 2$, is $L = \int_1^2 \sqrt{1 + \frac{16}{y^3}}\,dy$. 6) _____

A) $x = \frac{4}{\sqrt{y}}$ B) $x = \frac{-8}{\sqrt{y}}$ C) $x = -8\sqrt{y}$ D) $x = -8y^{5/2}$

Evaluate the improper integral or state that it is divergent. (Hint: u-sub)

7) $\int_0^\infty \frac{4(1 + \tan^{-1}x)}{1 + x^2}\,dx$ 7) _____

A) 2π B) $2\pi\left(1 + \frac{\pi}{4}\right)$ C) $2\left(1 + \frac{\pi}{2}\right)^2$ D) $4 \ln\left(1 + \frac{\pi}{2}\right)$

Evaluate the integral. (Hint: Substitution and Partial Fractions Decomposition)

8) $\int \frac{e^t\,dt}{e^{2t} - 7e^t + 6}$ 8) _____

A) $\frac{1}{5}\ln|e^t - 6| + \frac{1}{5}\ln|e^t - 1| + C$

B) $\frac{1}{5}\ln|e^t - 6| - \frac{1}{5}\ln|e^t - 1| + C$

C) $\frac{1}{5}e^t \ln|e^t - 6| - \frac{1}{5}e^t \ln|e^t - 1| + C$

D) $\frac{1}{5}\ln|t - 6| - \frac{1}{5}\ln|t - 1| + C$

Find the Maclaurin series for the given function.

9) e^{-7x} 9) _____

A) $\sum_{n=1}^{\infty} \frac{(-1)^n 7^n x^n}{n!}$

B) $\sum_{n=0}^{\infty} \frac{(-1)^n 7^n x^n}{n!}$

C) $\sum_{n=0}^{\infty} \frac{7^n x^n}{n!}$

D) $\sum_{n=1}^{\infty} \frac{7^n x^n}{n!}$

Provide an appropriate response.

10) Use the fact that $\cot x = \frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \ldots\right)$ for $|x| < \pi$ to find the first four terms of the series for $\ln(\sin x)$. 10) _____

A) $-\left(\frac{1}{x^2} + \frac{1}{3} + \frac{x^2}{15} + \frac{2x^4}{189} + \ldots\right)$

B) $-\ln|x| + \frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \ldots$

C) $\frac{1}{x^2} + \frac{1}{3} + \frac{x^2}{15} + \frac{2x^4}{189} + \ldots$

D) $\ln|x| - \left(\frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \ldots\right)$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question. Write your answer in the space provided. No partial credit.

Set up the integral to find the length of the curve. Simplify the integrand as much as you can. But DO NOT evaluate the integral.

11) $x = 3\sin^3 t$, $y = 3\cos^3 t$, $0 \le t \le \pi$ 11) _____

Find the value of d^2y/dx^2 at the point defined by the given value of t.

12) $x = \sqrt{t + 3}$, $y = -t$, $t = 13$ 12) _____

Evaluate the integral.

13) $\int x^4 \ln(4x)\, dx$ 13) _____

Determine the partial fractions decomposition to evaluate this integral. You just need to find the correct form of the decomposition. DO NOT solve for the coefficients or evaluate the integral.

14) $\int \frac{8x^2 + x + 63}{x^3 + 9x}\, dx$ 14) _____

ESSAY. Show all work to justify your answer. Answers with no work or insufficient work will receive no credit. Partial credit may be given.

Find the interval of convergence of the series.

15) $\sum_{n=0}^{\infty} \frac{(x - 4)^{2n}}{4^n}$

Use the integral test to determine whether the series converges.

16) $\sum_{n=1}^{\infty} \frac{\cos 1/n}{n^2}$

Solve the problem.

17) Find the volume of the solid generated by revolving the region bounded by the curve $y = \ln x$, the x-axis, and the vertical line $x = e^2$ about the x-axis.

Set up the integral to find the length of the curve. Simplify the integrand as much as you can. But DO NOT evaluate the integral.

18) The curve $r = 6\sqrt{1 + \cos 2\theta}$, $0 \le \theta \le \frac{\pi}{4}$

Solve the problem.

19) Using the Maclaurin series for $\tan^{-1} x$, obtain a series for $\frac{\tan^{-1}(x^2)}{x^2}$.

1) B
2) B
3) A
4) A
5) D
6) B
7) B
8) B
9) B
10) D
11) 9
12) -2
13) $\frac{1}{5}x^5 \ln 4x - \frac{1}{25}x^5 + C$
14) $7 \ln|x| + \frac{1}{2}\ln|x^2 + 9| + \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$
15) $2 < x < 6$
16) converges
17) $2\pi(e^2 - 1)$
18) $\frac{3}{2}\pi\sqrt{2}$
19) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{2n + 1}$