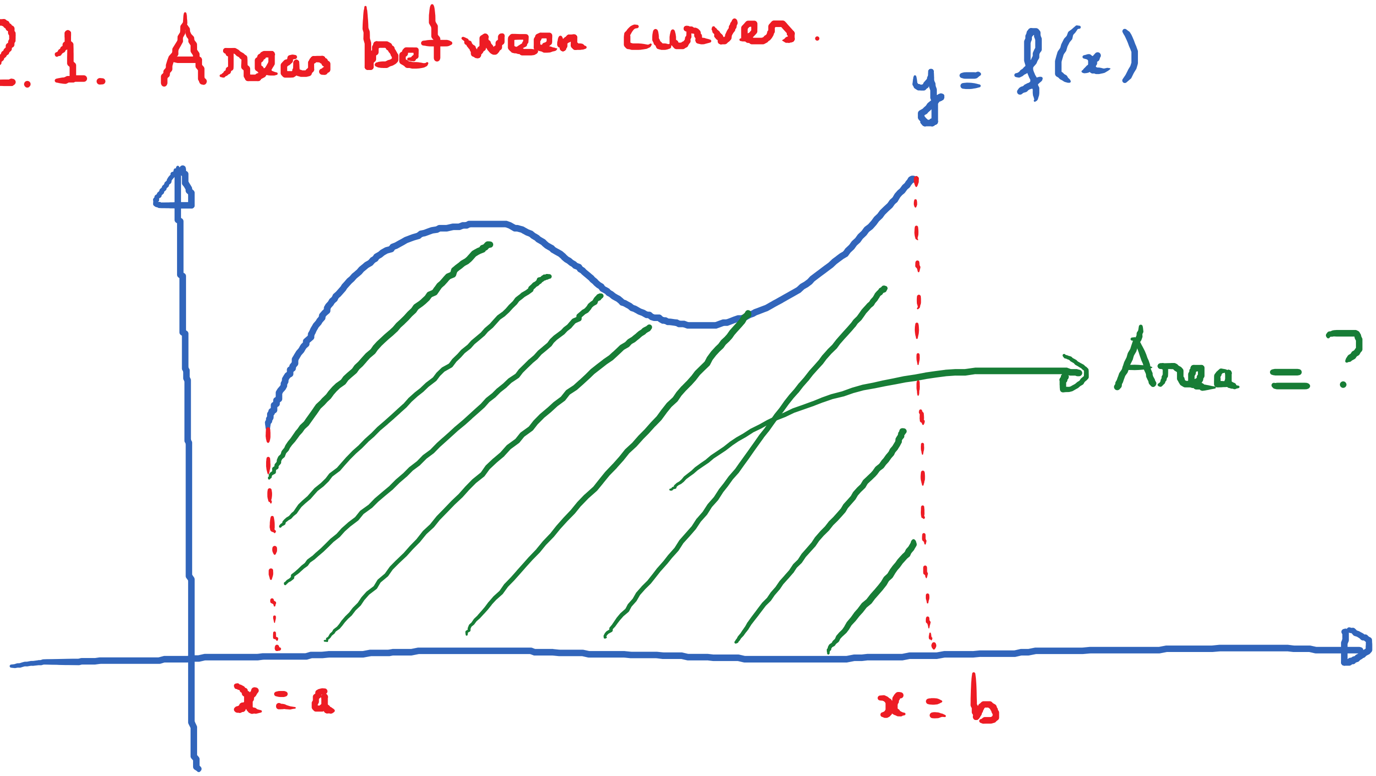
Class website:

apps.lonestar.edu/blogs/vindang.

Hover over Spring 2018. Select 2414

Dr. Dang.

2.1. Areas between curves.



$y = f(x)$; $a \le x \le b$

Find the area under the graph of $y = f(x)$;

$a \le x \le b$

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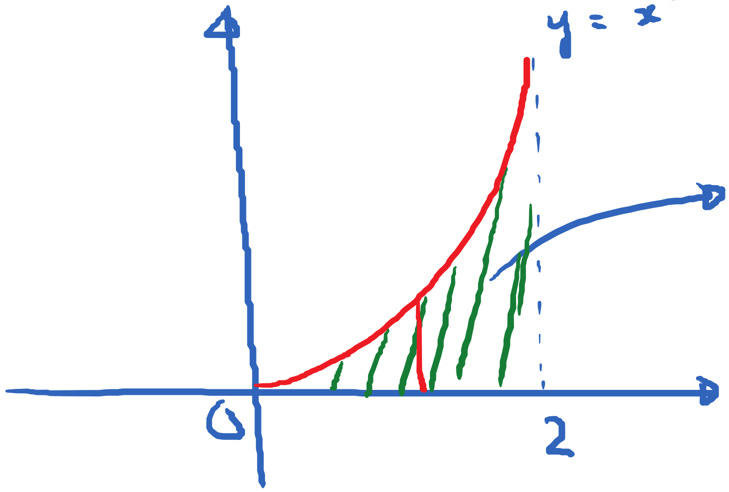


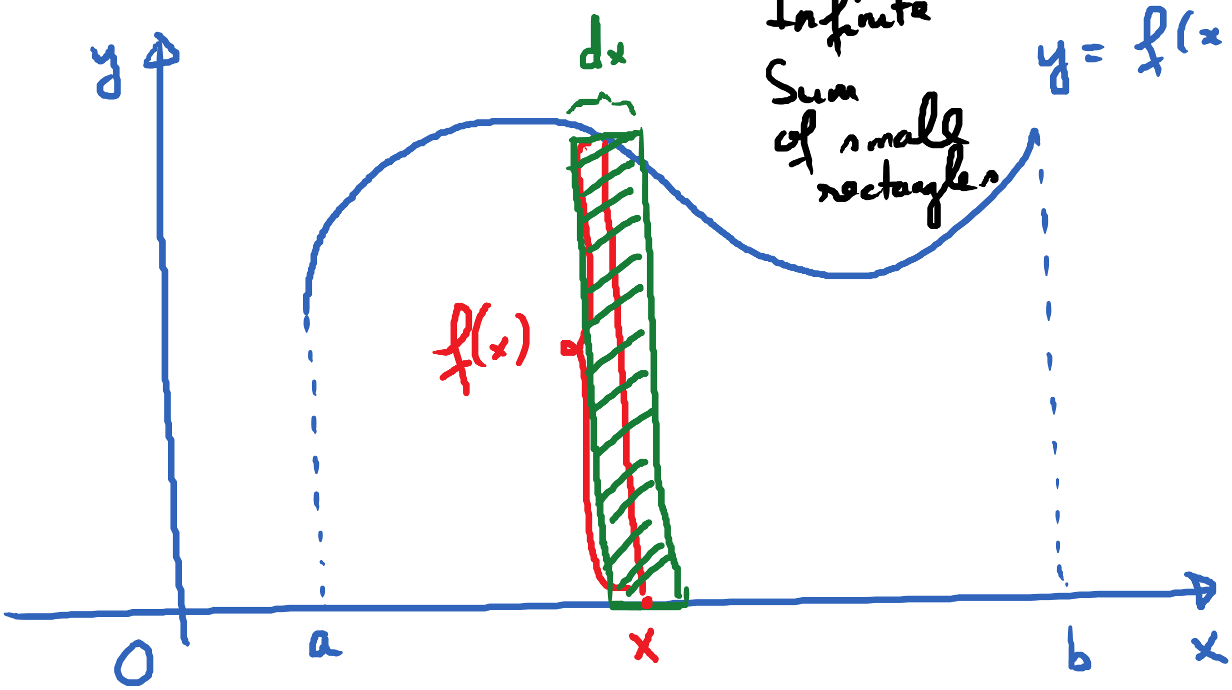
$$\text{Area} = \int_0^2 x^2\,dx$$

For the general situation :

area of a small rectangle

$$\boxed{\underset{a \le x \le b}{\text{Area under } f(x)} = \int_a^b \underbrace{f(x)}_{\text{height}}\ \underbrace{dx}_{\text{width}}}$$

Infinite Sum of small rectangles



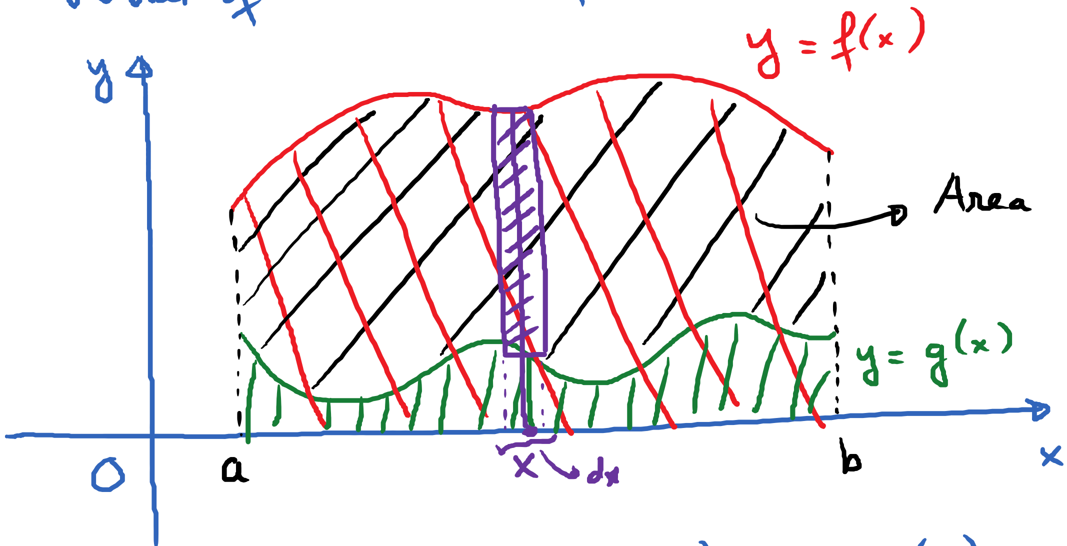
$$\text{Area} = \underbrace{\int_a^b}_{\text{Sum of areas}} \boxed{\underbrace{f(x)}_{\text{height}}\ \underbrace{dx}_{\text{width}}}$$

Area of small rectangle

height width of small rectangle

Sum of areas

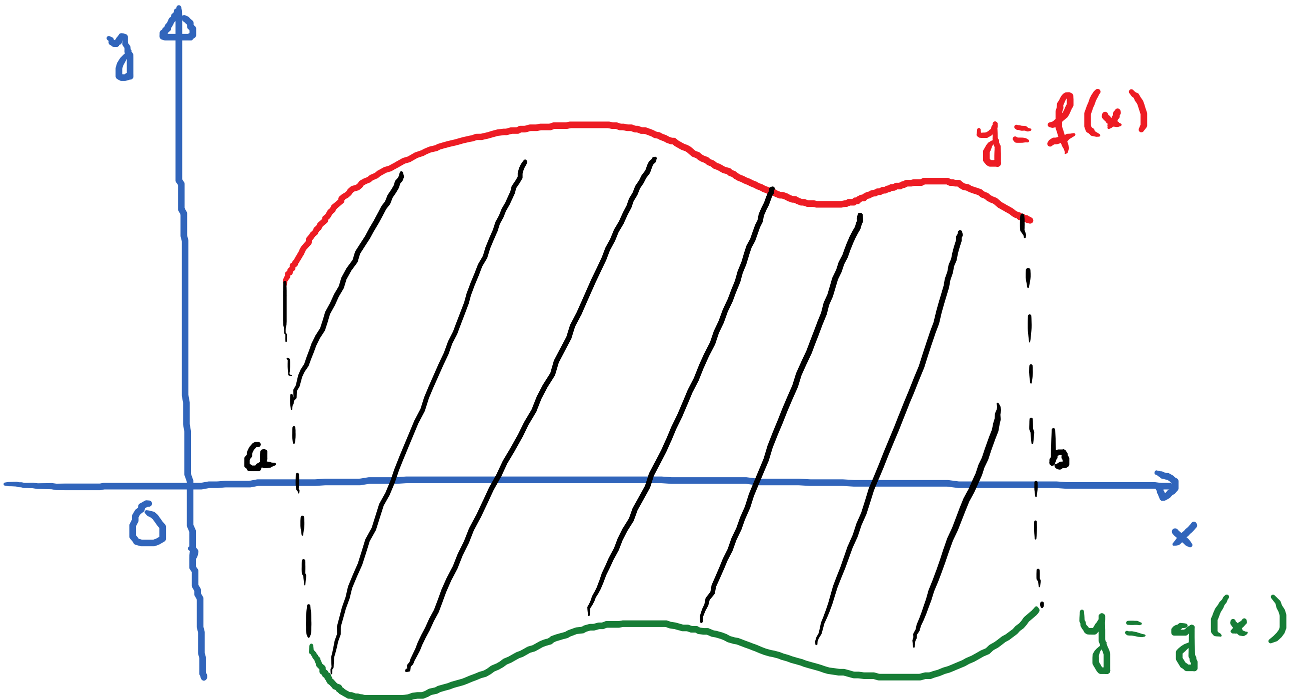
What if we have 2 functions?



Find area bounded by $y = f(x)$; $y = g(x)$;

$a \leq x \leq b$

$$\text{Area} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \boxed{\left(f(x) - g(x)\right)}\,dx$$



$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b (f(x) - g(x))\,dx$$

Area of the region bounded by 2 functions:

$$A = \int_a^b (\text{top function} - \text{bottom function})\,dx$$

Ex 1 (HW).

1st : Find points of intersection.

$$x^3 - 2x^2 + 3 = 3x^2 + 5x - 22.$$

$$\underbrace{x^3 - 5x^2}\ \underbrace{-\ 5x + 25} = 0$$

$$x^2(x-5) - 5(x-5) = 0$$

$$(x-5)(x^2-5) = 0$$

$x = 5$ or $x = \pm\sqrt{5}$

From picture, the x-coordinates of the points of intersection are $x = -\sqrt{5}$ and $x = \sqrt{5}$

2nd : Area $= \displaystyle\int_{-\sqrt{5}}^{\sqrt{5}} \left(x^3 - 5x^2 - 5x + 25\right) dx$

$$= \left(\frac{x^4}{4} - 5\frac{x^3}{3} - 5\frac{x^2}{2} + 25x\right)\Bigg|_{-\sqrt{5}}^{\sqrt{5}}$$

→ Plug in & simplify : $\boxed{\dfrac{100\sqrt{5}}{3}}$

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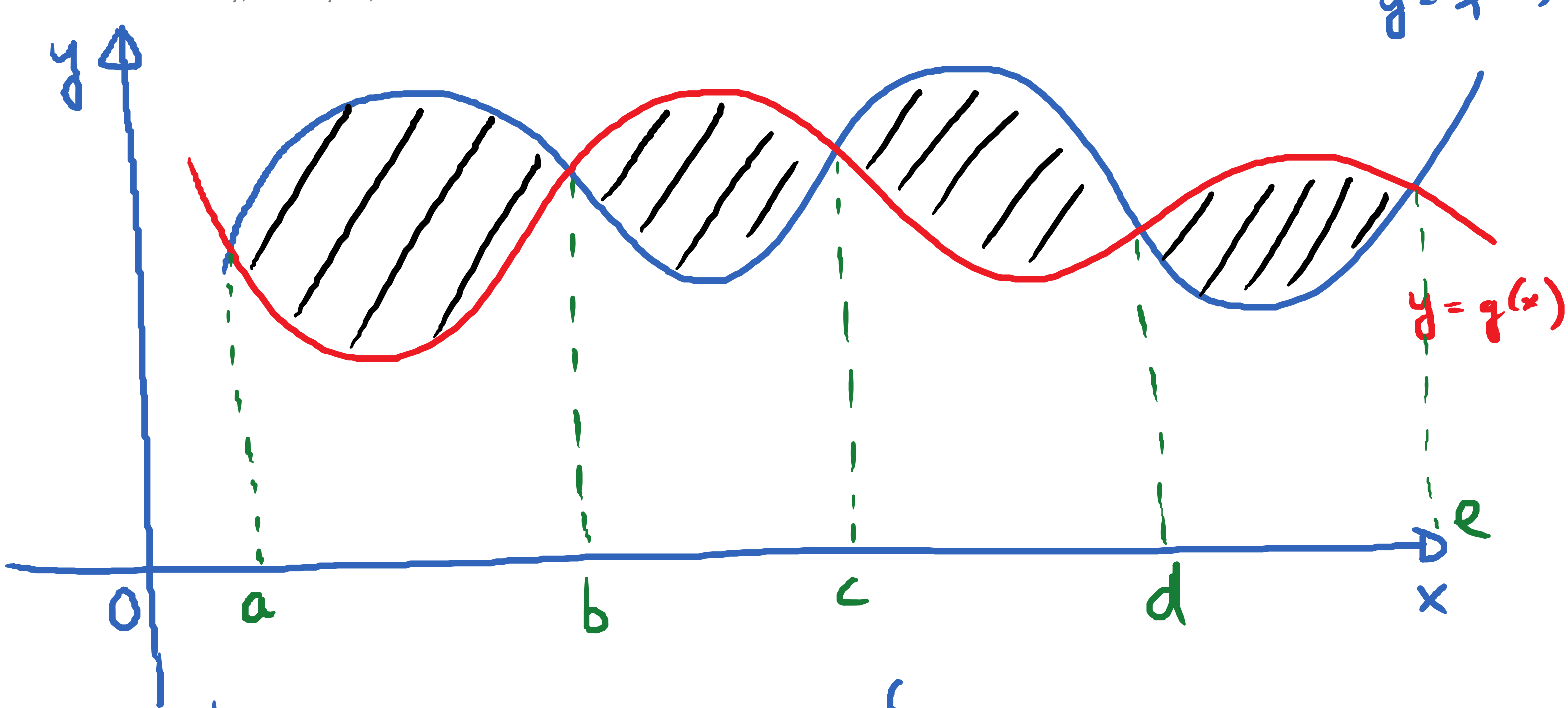
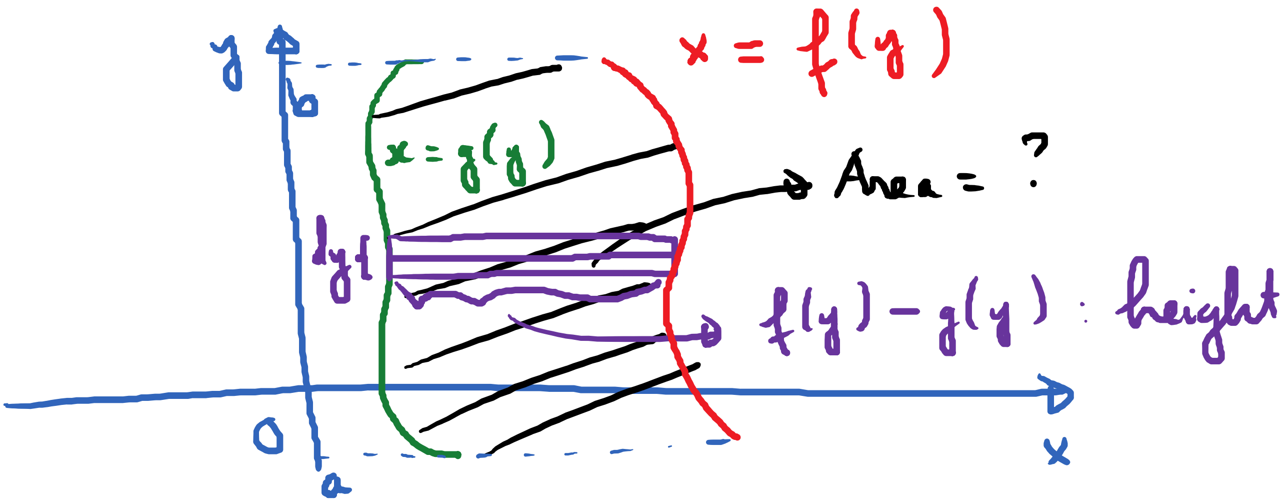


$$\int_a^b \big(f(x)-g(x)\big)\,dx + \int_b^c \big(g(x)-f(x)\big)\,dx + \int_c^d \big(f(x)-g(x)\big)\,dx + \int_d^e \big(g(x)-f(x)\big)\,dx$$



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$$A = \int_a^b \left(\underbrace{f(y)}_{\text{Rightmost}} - \underbrace{g(y)}_{\text{leftmost}} \right) dy$$

Ex 13. (HW 1)

Find points of intersection:

$$-5 + y^2 = 3y - y^2 \qquad 2\cdot(-5) = -10$$

$$2y^2 - 3y - 5 = 0$$

$$2y^2 + 2y - 5y - 5 = 0$$

$$2y(y+1) - 5(y+1) = 0$$

$$(2y-5)(y+1) = 0$$

$$y = -1 \; ; \; y = \frac{5}{2}.$$

$$\int_{-1}^{5/2} \left[(3y - y^2) - (-5 + y^2) \right] dy$$

$$\int_{-1}^{5/2} \left(-2y^2 + 3y + 5\right) dy$$

$$\left(-2\frac{y^3}{3} + 3\frac{y^2}{2} + 5y\right)\Bigg|_{-1}^{5/2}$$

$$-2\cdot\frac{\left(\frac{5}{2}\right)^3}{3} + 3\cdot\frac{\left(\frac{5}{2}\right)^2}{2} + 5\cdot\left(\frac{5}{2}\right) =$$

$$-\left(-2\,\frac{(-1)^3}{3} + 3\cdot\frac{(-1)^2}{2} + 5\cdot(-1)\right)$$