Practice 1 – Essay

Thursday, February 8, 2018 11:59 AM

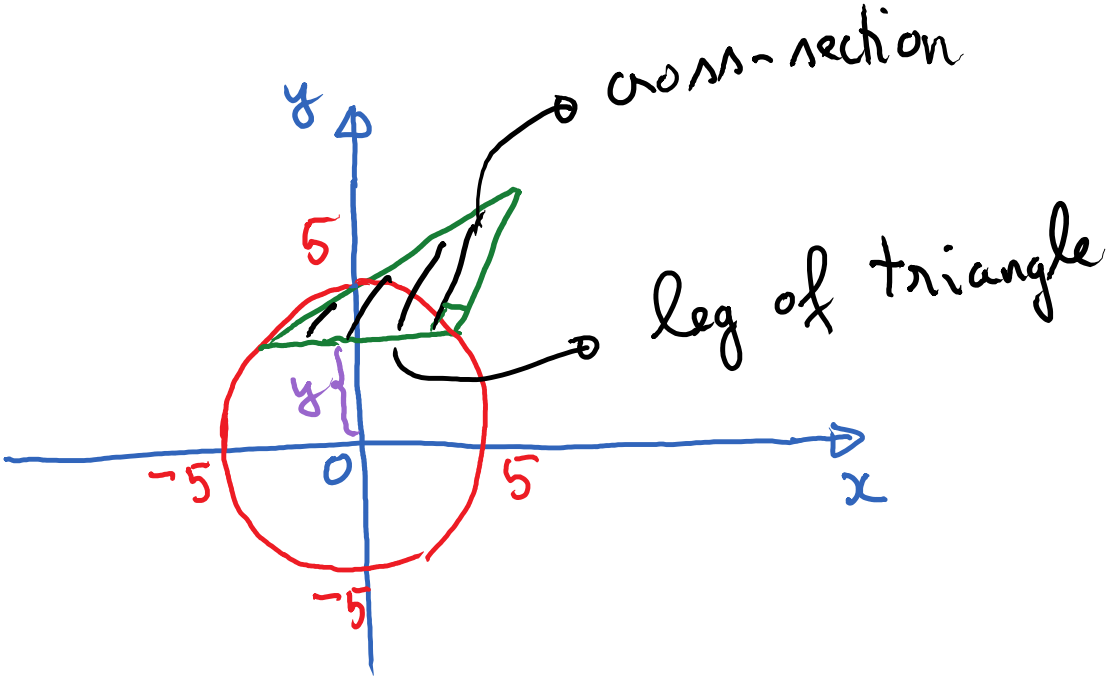
#13.

$$\text{Area} = \int_0^2 \left(\frac{1}{2}x + 1 - e^{-x}\right) dx$$

$$= \left(\frac{x^2}{4} + x + e^{-x}\right)\Big|_0^2$$

$$= 3 + e^{-2} - e^0 = \boxed{2 + e^{-2}}$$

14.



Area of a cross section at y is:

$$A(y) = \frac{1}{2}(\text{leg})^2 = \frac{1}{2}(2g(y))^2$$

Since $x^2 + y^2 = 25$; $x = \pm\sqrt{25 - y^2}$.

So, we can choose $g(y) = \sqrt{25 - y^2}$

Thus, $A(y) = \frac{1}{2} \cdot \left(2\sqrt{25 - y^2}\right)^2$

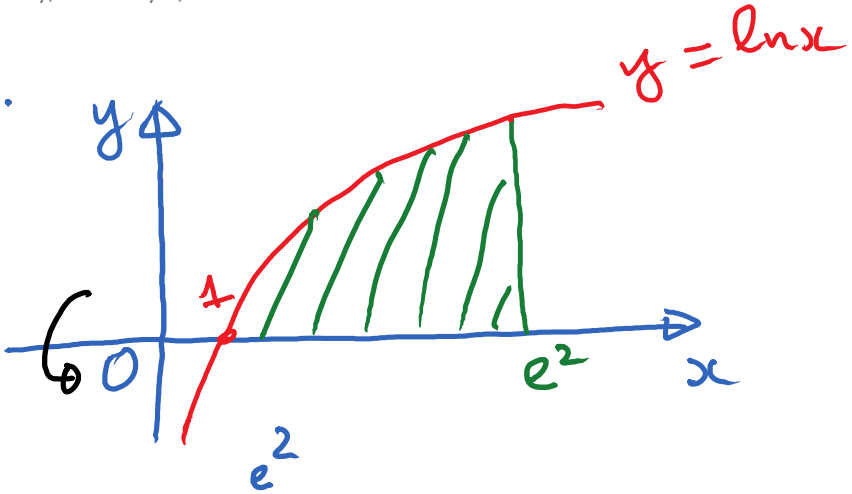
$$A(y) = 2(25 - y^2) = 50 - 2y^2$$

$$\text{Volume Solid} = \int_{-5}^{5} A(y)\,dy = \int_{-5}^{5} (50 - 2y^2)\,dy$$

$$= \left(50y - \frac{2y^3}{3}\right)\Bigg|_{-5}^{5}$$

$$= \boxed{\frac{1000}{3}}$$

#15.



$$\text{Volume} = \pi \int_1^{e^2} (\ln x)^2 \, dx$$

$$\text{let } u = (\ln x)^2 \quad ; \quad du = \frac{2 \ln x}{x}$$

$$dv = dx \qquad v = x$$

$$V = \pi \cdot \left[x (\ln x)^2 \Big|_1^{e^2} - \int_1^{e^2} x \cdot \frac{2 \ln x}{x} \, dx \right]$$

$$= \pi \cdot \left[4e^2 - 2 \int_1^{e^2} \ln x \, dx \right]$$

Now, we find $\int_1^{e^2} \ln x \, dx$

Let $u = \ln x$. $\quad du = \frac{1}{x}$

$dv = dx \quad v = x$

$$\int_1^{e^2} \ln x \, dx = x \ln x \Big|_1^{e^2} - \int_1^{e^2} x \cdot \frac{1}{x} dx$$

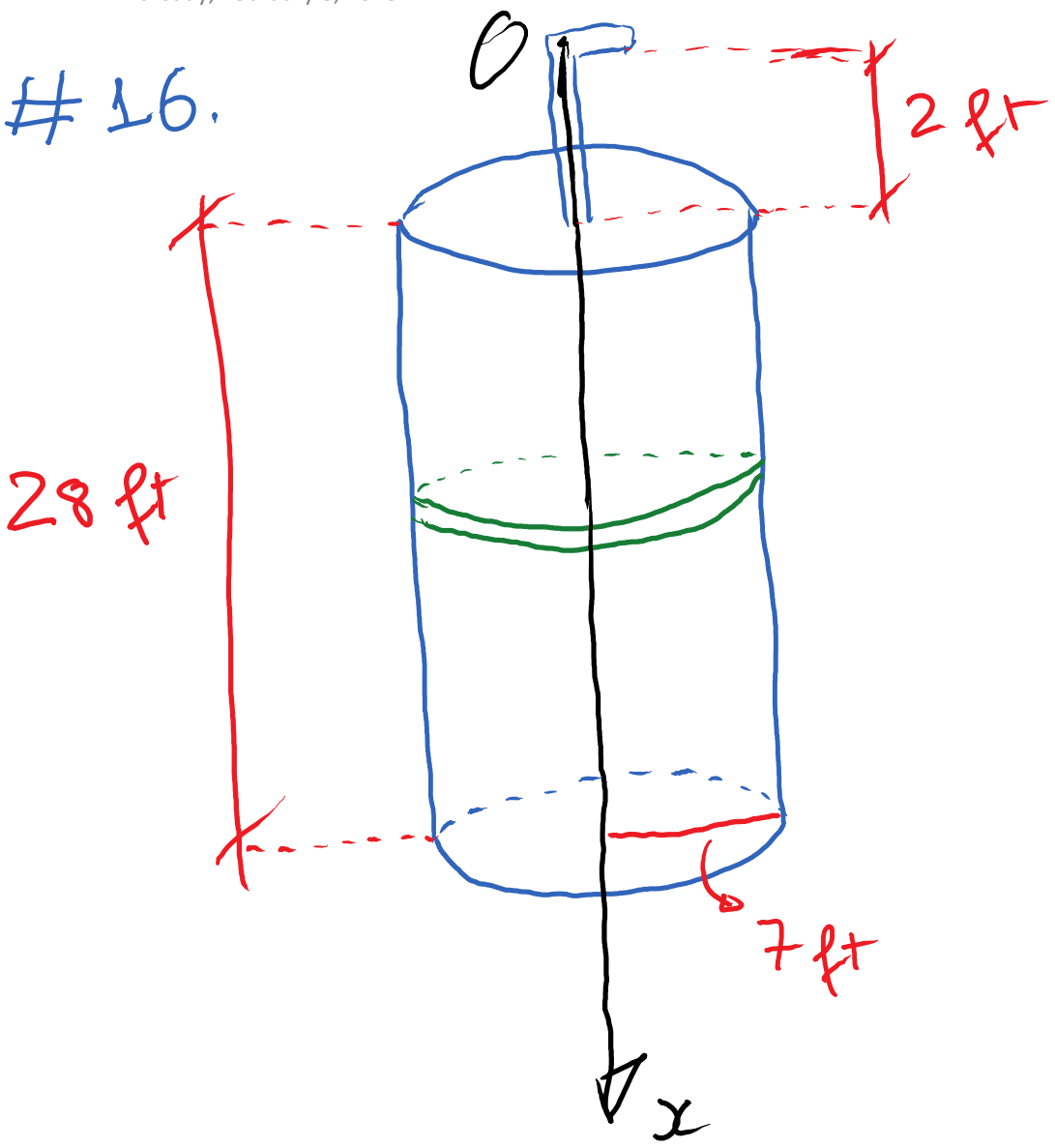
$$= 2e^2 - \int_1^{e^2} dx = 2e^2 - x \Big|_1^{e^2}$$

$$= e^2 + 1$$

So, $V = \pi \left[4e^2 - 2(e^2 + 1) \right]$

$$= \pi \left(2e^2 - 2 \right) = \boxed{2\pi \left(e^2 - 1 \right)}$$

#16.



Force required to lift a slice at x

$= (\text{Volume}_{\text{slice}}) \cdot (\text{density})$

$= (\pi \cdot (7)^2 \cdot dx)(60) = 2940\pi\, dx$

Distance to move slice $= x$

Work done in lifting a slice at $x = 2940\pi x\,dx$

Work done in pumping the entire volume of water:

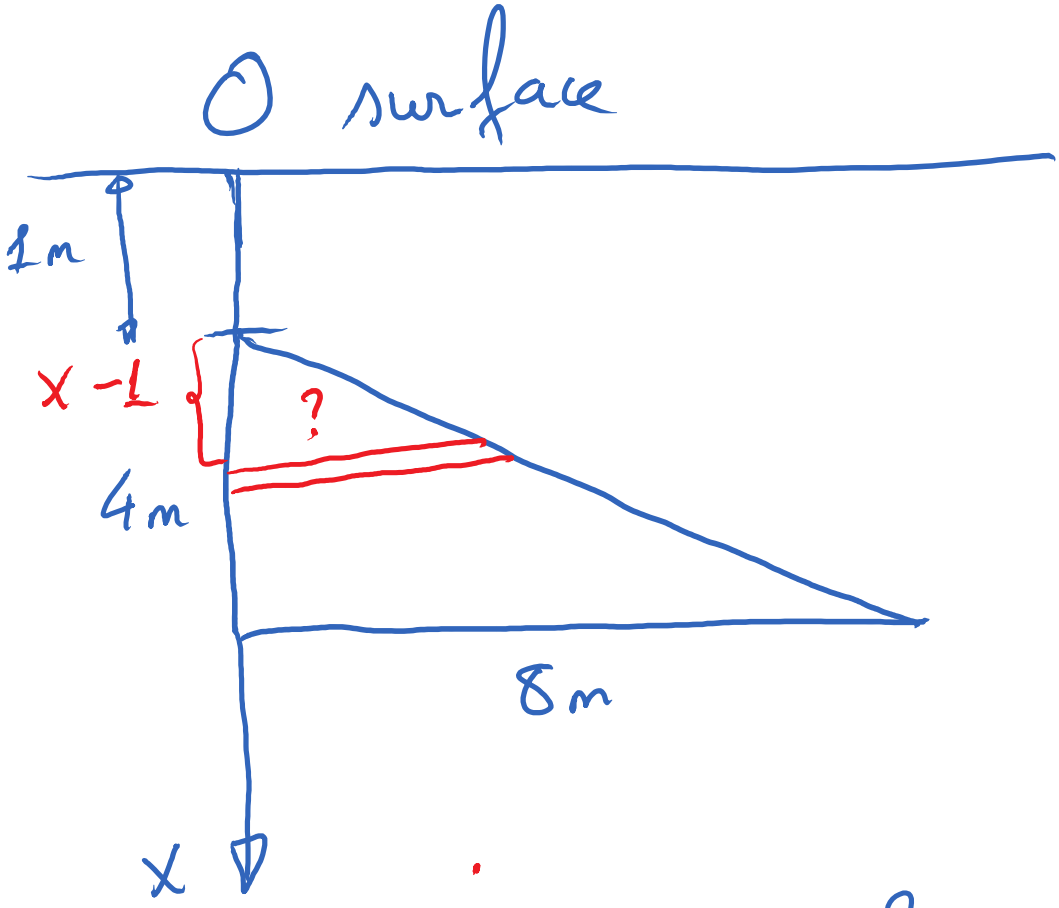
$$W = \int_2^{30} 2940\pi x\,dx$$

$$= 2940\pi \cdot \frac{x^2}{2}\Big|_2^{30}$$

$$\approx 4137854.52$$

For this kind of problem on the test, it is enough to set up this integral, without evaluating it.

#17.



The goal is to find the force on a horizontal strip and apply integration to find the force on the object.

$$\text{Force}_{\text{Strip}} = (\text{density}) \cdot (\text{area}) \cdot (\text{depth})$$

density $= 9800\ N/m^3$ (given)

depth of a strip at $x = x$ (By choice of origin O)

Area = length · width = (?) · dx

Need to find (?)

By similar triangles:

$$\frac{?}{8} = \frac{x-1}{4}.$$

So, $? = 8 \cdot \left(\frac{x-1}{4}\right) = 2(x-1)$

So, $\text{Force}_{\text{strip}} = (9800) \cdot 2(x-1) \cdot x \cdot dx$

$= 19600(x-1)x\,dx$

Force on object

$$= \int_1^5 19600(x-1)\,x\,dx$$