Practice Test 3 - Calculus II - Spring 2018

MULTIPLE CHOICE. (5pts each) Choose the one alternative that best completes the statement or answers the question. Write your choice in the space provided. No partial credit.

Provide an appropriate response.

1) For an alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} u_n$ where it is not true that $u_n \geq u_{n+1}$ for sufficiently large n, what can be said about the convergence or divergence of the series? 1) ________

A) The series always converges.
B) The series always diverges.
C) The series may or may not converge.

Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series.

2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0.1)^{2n+1}}{2n+1}$ 2) ________

A) 1.11×10^{-10} B) 9.09×10^{-13} C) 8.33×10^{-14} D) 1.00×10^{-11}

Use the root test to determine if the series converges or diverges.

3) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{3n+7}\right)^n$ 3) ________

A) Diverges B) Converges

Find the series' radius of convergence.

4) $\sum_{n=0}^{\infty} \frac{n(n+1)(n+2)}{6^n}(x-\pi)^n$ 4) ________

A) 1 B) 6 C) ∞, for all x D) 12

For what values of x does the series converge absolutely?

5) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n^3+6}}$ 5) ________

A) $-1 \leq x \leq 1$ B) $-1 < x \leq 1$ C) $-1 \leq x < 1$ D) $-1 < x < 1$

Find the sum of the series as a function of x.

6) $\sum_{n=0}^{\infty} \left(\frac{x^2+4}{3}\right)^n$ 6) ________

A) $\frac{3}{x^2-1}$ B) $\frac{3}{x^2+1}$ C) $-\frac{3}{x^2-1}$ D) $-\frac{3}{x^2+1}$

Find the Maclaurin series for the given function.

7) e^{-7x} 7) ________

A) $\sum_{n=1}^{\infty} \frac{7^n x^n}{n!}$

B) $\sum_{n=0}^{\infty} \frac{7^n x^n}{n!}$

C) $\sum_{n=1}^{\infty} \frac{(-1)^n 7^n x^n}{n!}$

D) $\sum_{n=0}^{\infty} \frac{(-1)^n 7^n x^n}{n!}$

Use power series operations to find the Taylor series at x = 0 for the given function.

8) $f(x) = x^8 \sin x$ 8) ________

A) $\sum_{n=0}^{\infty} \frac{(-1)^n 8^n x^{2n}}{(2n + 9)!}$

B) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+9}}{(2n + 1)!}$

C) $\sum_{n=0}^{\infty} \frac{(-1)^n 8^n x^{2n}}{(2n + 1)!}$

D) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+9}}{(2n + 9)!}$

Provide an appropriate response.

9) Use the fact that $\cot x = \frac{1}{x} - \left(\frac{x}{3} + \frac{x^3}{45} + \frac{2x^5}{945} + \ldots\right)$ for $|x| < \pi$ to find the first four terms of the series for ln(sin x). 9) ________

A) $\ln|x| - \left(\frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \ldots\right)$

B) $-\left(\frac{1}{x^2} + \frac{1}{3} + \frac{x^2}{15} + \frac{2x^4}{189} + \ldots\right)$

C) $-\ln|x| + \frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \ldots$

D) $\frac{1}{x^2} + \frac{1}{3} + \frac{x^2}{15} + \frac{2x^4}{189} + \ldots$

Find the sum of the series.

10) $\frac{2}{3} - \frac{2^3}{3^3 \cdot 3!} + \frac{2^5}{3^5 \cdot 5!} - \frac{2^7}{3^7 \cdot 7!} + \ldots$ 10) ______

A) $\cos \frac{2}{3}$

B) $\ln \frac{2}{3}$

C) $\sin \frac{2}{3}$

D) $\sin \frac{3}{2}$

SHORT ANSWER. (5pts each) Write the word or phrase that best completes each statement or answers the question. Write your answer in the space provided. No partial credit.

Determine convergence or divergence of the alternating series.

11) $\sum_{n=1}^{\infty} (-1)^n \ln\left[\frac{6n + 3}{6n + 2}\right]$ 11) ______________

Provide an appropriate response.

12) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{7^{n-1}}$ by expressing $\frac{1}{1 - x}$ as a geometric series, differentiating both sides of the resulting equation with respect to x, and replacing x by $\frac{1}{7}$. 12) ______________

Find the Maclaurin series for the given function.

13) $\dfrac{\sin 5x}{x}$ 13) ______________

Solve the problem.

14) Using the Maclaurin series for $\ln(1 + x)$, obtain a series for $\dfrac{\ln(1 + x^2)}{x}$. 14) ______________

ESSAY. (6pts each) Show all work to justify your answer. Answers with no work or insufficient work will receive no credit. Partial credit may be given.

Use the ratio test to determine if the series converges or diverges.

15) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{3^n(n!)^2}$

Find the interval of convergence of the series.

16) $\displaystyle\sum_{n=0}^{\infty} \frac{(x - 4)^{2n}}{4^n}$

Use the Error Estimate for Alternating Series to find the number of terms necessary to approximate sum accurate to three decimal places, then calculate the approximation.

17) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(4n+1)^3}$

Find the Taylor polynomial of order 3 generated by f at a.

18) $f(x) = \ln x$, $a = 10$

Provide an appropriate response.

19) Derive a series for $\ln(1 + x^2)$ for $x > 1$ by first finding the series for $\frac{x}{1 + x^2}$ and then integrating.

$\left(\text{Hint: } \frac{x}{1 + x^2} = \frac{1}{x} \frac{1}{1 + 1/x^2}\right)$