Note: If we rename f^{-1} as g in the formula for the I.F.T., then we have:

f, g are inverse functions of each other

$$\Longrightarrow \boxed{g'(x) = \frac{1}{f'(g(x))}}$$

→ We are ready to derive the formulas for the derivative of $y = \arcsin(x)$; $y = \arccos(x)$, $y = \arctan(x)$, etc.

E.g. let $f(x) = \tan(x)$ and $g(x) = \boxed{\arctan(x)}$

Goal: Find a formula for $g'(x)$

Note: f and g are inverse functions of each other.

→ We can apply the I.F.T.

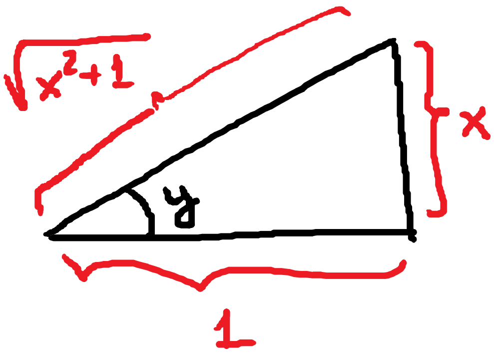
$$g'(x) = \frac{1}{f'(\boxed{g(x)})}$$

$$g'(x) = \frac{1}{f'(\arctan(x))} = \frac{1}{\underbrace{\sec^2(\arctan(x))}_{\frac{1}{\sec^2(y)}}}$$

$f(x) = \tan(x) \longrightarrow f'(x) = \sec^2(x)$

Goal: Simplify

let $\arctan(x) = y$. Then $\boxed{\tan(y) = x}$



$\tan = \frac{\text{opp}}{\text{adj}}$

So, $\sec(y) = \frac{\text{hyp.}}{\text{adj.}} = \frac{\sqrt{x^2+1}}{1}$

$= \sqrt{x^2+1}$

So, $g'(x) = \frac{1}{\sec^2(y)} = \frac{1}{x^2+1}$

So, we derived the formula for $\frac{d}{dx}(\arctan(x))$, that is,

$$\boxed{\frac{d}{dx}(\arctan(x)) = \frac{1}{x^2+1}}$$

Ex. Derive the formulas for.

1. $$\frac{d}{dx}\left(\arcsin(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

2. $$\frac{d}{dx}\left(\arccos(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

To sum up,

$$\boxed{\begin{aligned} \frac{d}{dx}\left(\arcsin(x)\right) &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\left(\arccos(x)\right) &= \frac{-1}{\sqrt{1-x^2}} \\ \frac{d}{dx}\left(\arctan(x)\right) &= \frac{1}{x^2+1} \end{aligned}}$$

If u is a function of x, then

$$\boxed{\begin{aligned} \frac{d}{dx}\left(\arcsin(u)\right) &= \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \\ \frac{d}{dx}\left(\arccos(u)\right) &= \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} \\ \frac{d}{dx}\left(\arctan(u)\right) &= \frac{1}{1+u^2} \cdot \frac{du}{dx} \end{aligned}}$$

Ex. 1 Find $\frac{dy}{dx}$ of the given function

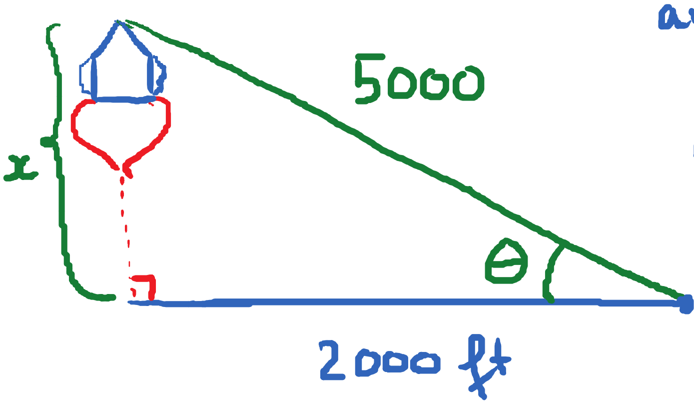
a) $y = \arctan(x^2) \longrightarrow \frac{dy}{dx} = \frac{2x}{1+x^4}$

b) $y = \arccos(3x-1) \longrightarrow \frac{dy}{dx} = \frac{-3}{\sqrt{1-(3x-1)^2}}$

c) $y = x^2 \cdot \arcsin(x)$

$$\longrightarrow \frac{dy}{dx} = 2x \cdot \arcsin(x) + \frac{x^2}{\sqrt{1-x^2}}$$

(2)



Television camera is located 2000 ft away from the launching pad of a rocket.

Let x be the height of the rocket.

(a) Write θ as a function of x. (θ as in picture)

(b) Find $\dfrac{d\theta}{dx}$. Evaluate it when the rocket is 5000 ft from the camera.

(a) $\tan\theta = \dfrac{x}{2000} \longrightarrow \theta = \arctan\left(\dfrac{x}{2000}\right)$

(b) $$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{x}{2000}\right)^2}\cdot\frac{1}{2000}$$

When rocket is 5000 ft to camera,

$$x = \sqrt{(5000)^2 - (2000)^2} = \cdots$$

Then plug x into $\dfrac{d\theta}{dx}$.