3.8. Implicit Differentiation

Wednesday, July 25, 2018 10:08 AM

Goal: To find derivatives implicitly.

So far, we are given $y =$ formula in x (explicitly)

E.g. $y = x^3 + 2x - 3 \longrightarrow$ find $\frac{dy}{dx}$

$\longrightarrow$ This is explicit differentiation

$$\frac{dy}{dx} = 3x^2 + 2$$

In many situations, y is not given explicitly in terms of x.

E.g. Given the equation $x^2 + y^2 = 25$

$\longrightarrow$ y does depend on x.

$\longrightarrow$ It makes sense to ask for $\frac{dy}{dx}$.

But here, we don't have a formula $y = \dots$ (something in x).

The method of finding $\frac{dy}{dx}$ without having to get y by itself first is called implicit differentiation.

Chain Rule is very important here.

To take the derivative of a composition of functions, we take the derivative of the outside and multiply it by the derivative of the inside.

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

outside

$$\frac{d}{dx}\left[\boxed{u}^{\boxed{2}}\right] = 2u \cdot \frac{du}{dx}$$

inside

$$\frac{d}{dx}\left[\boxed{\sin}(\boxed{u})\right] = \cos(u) \cdot \frac{du}{dx}$$

outside inside

E.g. to illustrate implicit differentiation.

Given that $x^2 + y^2 = 25$. Find $\frac{dy}{dx}$?

1st Step: Differentiate both sides with respect to x.

$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[25]$$

outside

$$\frac{d}{dx}[x^2] + \frac{d}{dx}\left[\boxed{y}^{\boxed{2}}\right] = 0$$

inside

power rule + chain rule

$$2x + 2y \cdot \boxed{\frac{dy}{dx}} = 0$$

→ Goal: get $\frac{dy}{dx}$ by itself.

$$2y \cdot \frac{dy}{dx} = -2x$$

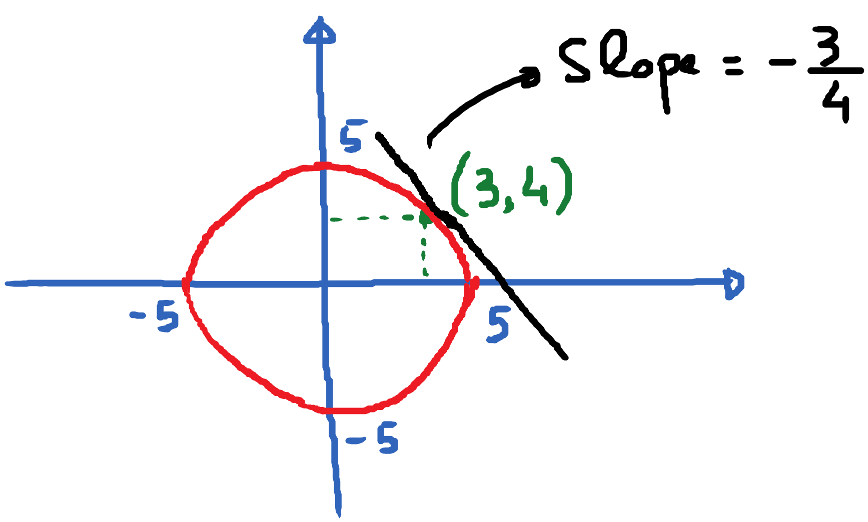
$$\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}.$$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

* Find the derivative $\frac{dy}{dx}$ at the point $(3, 4)$?

$$\left.\frac{dy}{dx}\right|_{x=3;\, y=4} = -\frac{3}{4}.$$

* Equation: $x^2 + y^2 = 25$ → this is the equation for a circle centered at $(0,0)$ with radius 5.



Ex. Given $4x^5 + \tan(y) = y^2 + 5x$.

Find $\frac{dy}{dx}$?

$$\rightarrow \frac{d}{dx}\left[4x^5 + \tan(y)\right] = \frac{d}{dx}\left[y^2 + 5x\right]$$

$$\rightarrow \frac{d}{dx}\left[4x^5\right] + \frac{d}{dx}\left[\tan(y)\right] = \frac{d}{dx}\left[y^2\right] + \frac{d}{dx}\left[5x\right]$$

$$\rightarrow 20x^4 + \sec^2(y)\cdot\frac{dy}{dx} = 2y\cdot\frac{dy}{dx} + 5$$

$$\rightarrow \sec^2(y)\cdot\frac{dy}{dx} - 2y\cdot\frac{dy}{dx} = 5 - 20x^4$$

$$\rightarrow \left[\sec^2(y) - 2y\right]\cdot\frac{dy}{dx} = 5 - 20x^4$$

$$\rightarrow \boxed{\frac{dy}{dx} = \frac{5 - 20x^4}{\sec^2(y) - 2y}}$$