Ex. Given the equation: $x^3y + xy^3 = -8$.

Find $\dfrac{dy}{dx}$?

$$\rightarrow \underbrace{\frac{d}{dx}[x^3y]}_{\text{product rule}} + \underbrace{\frac{d}{dx}[xy^3]}_{\text{product rule}} = 0$$

$$\frac{d}{dx}[x^3]\cdot y + x^3\cdot\frac{dy}{dx} + \frac{d}{dx}[x]\cdot y^3 + x\cdot\underbrace{\frac{d}{dx}[y^3]}_{\text{power + chain}} = 0$$

$$3x^2y + \boxed{x^3\frac{dy}{dx}} + y^3 + \boxed{x\cdot 3y^2\cdot\frac{dy}{dx}} = 0$$

$$(x^3 + 3xy^2)\frac{dy}{dx} = -3x^2y - y^3$$

$$\boxed{\frac{dy}{dx} = -\left(\frac{3x^2y + y^3}{x^3 + 3xy^2}\right)}$$

E.g. Given the equation:

$$x^2 + y^2 = \left(2x^2 + 2y^2 - x\right)^2.$$

This equation represents a curve called the Cardioid.

Q: Find the equation of the tangent line to this curve at the point $\left(0, \frac{1}{2}\right)$.

Need Slope = derivative.

Find $\frac{dy}{dx}$.

outside

inside

$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}\left[\left(\boxed{2x^2 + 2y^2 - x}\right)^{\boxed{2}}\right]$$

$$2x + 2y \cdot \frac{dy}{dx} = 2\left(2x^2 + 2y^2 - x\right) \cdot \frac{d}{dx}\left(2x^2 + 2y^2 - x\right)$$

$$2x + 2y \cdot \frac{dy}{dx} = 2\left(2x^2 + 2y^2 - x\right) \cdot \left(4x + 4y\frac{dy}{dx} - 1\right)$$

Plug in $x = 0$; $y = \frac{1}{2}$ to find $\frac{dy}{dx}$ at that particular point:

$$\frac{dy}{dx} = 2 \cdot \frac{1}{2} \cdot \left(2\frac{dy}{dx} - 1\right)$$

$$\frac{dy}{dx} = 2 \cdot \frac{dy}{dx} - 1$$

$$-\frac{dy}{dx} = -1 \rightarrow \frac{dy}{dx} = 1 \leftarrow \text{Slope.}$$

Equation of tangent line: $\boxed{y = x + \frac{1}{2}}$.

Find $\frac{d^2y}{dx^2}$ implicitly.

E.g. Given the equation $x^2 + y^2 = 25$.

Find $\frac{d^2y}{dx^2}$.

Wednesday, July 25, 2018 11:04 AM

We already found $\boxed{\dfrac{dy}{dx} = -\dfrac{x}{y}.}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{x}{y}\right)$$

$$= -\frac{d}{dx}\left(\frac{\overset{\text{high}}{\boxed{x}}}{\underset{\text{Low}}{\boxed{y}}}\right) = -\frac{y \cdot 1 - x \cdot \boxed{\frac{dy}{dx}}}{y^2}$$

quotient rule

$$= -\frac{y - x \cdot \left(-\frac{x}{y}\right)}{y^2}$$

$$= -\frac{y + \frac{x^2}{y}}{y^2} = -\frac{\frac{y^2 + x^2}{y}}{\frac{y^2}{1}}$$

$$= \boxed{-\frac{y^2 + x^2}{y^3}} \longrightarrow \text{second derivative w.r.t. } x$$