3.9. Derivatives of Exponential and Log Functions

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$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u . \frac{du}{dx}$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arcsin u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$$

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Derivatives of Exponential Functions

Base e: $e \approx 2.71828$

$f(x) = e^x$; $f(0) = e^0 = 1$; $f(1) = e$; $f(2) = e^2$

$f(-1) = \frac{1}{e}$, etc.

Domain: $(-\infty, \infty)$

If $f(x) = e^x$, then $f'(x) = e^x$

In Leibnitz notation: $\boxed{\frac{d}{dx}\left[e^x\right] = e^x}$

If u is a function of x, $\boxed{\frac{d}{dx}\left[e^u\right] = e^u \cdot \frac{du}{dx}}$.

In Newton's notation:

$$\boxed{\left(e^{f(x)}\right)' = e^{f(x)} \cdot f'(x)}.$$

Ex. Find the derivative:

(a) $\frac{d}{dx}\left[e^{2018}\right] = 0$ (b) $\underbrace{\frac{d}{dx}\left[e^{-x}\right]}_{-e^{-x}}$ (c) $\underbrace{\frac{d}{dx}\left[e^{\sec x}\right]}_{e^{\sec x} \cdot \sec x \tan x}$

(d) $\frac{d}{dx}\left[\frac{x}{e^{2x}}\right] = \frac{1-2x}{e^{2x}}$

2 Derivatives of the natural log function.

The natural log function is the function

$$f(x) = \ln x.$$

$\left(\ln 1 = 0 \,;\ \ln e = 1 \,;\ \ln e^2 = 2 \,;\right.$

$\left. \text{in general } \ln(e^x) = x \right)$

Formula for the derivative of $f(x) = \ln x$ is:

$$\boxed{\frac{d}{dx}(\ln x) = \frac{1}{x}} \qquad \boxed{\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}}$$

$$\left(\frac{d}{dx}(\ln u) = \frac{u'}{u} \right)$$

$$\boxed{\left(\ln(f(x))\right)' = \frac{f'(x)}{f(x)}}$$

Why is $\frac{d}{dx}(\ln x) = \frac{1}{x}$?

$y = \ln x$. <u>Goal:</u> Develop a formula for $\frac{dy}{dx}$?

$\longleftrightarrow e^{y} = x$

Use implicit differentiation to take the derivative of both sides of the above equation w.r.t. x

$$\frac{d}{dx}\left[e^{y}\right] = \frac{d}{dx}\left[x\right]$$

$$e^{y} \cdot \boxed{\frac{dy}{dx}} = 1$$

$$\frac{dy}{dx} = \frac{1}{e^{y}} = \frac{1}{x}.$$

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E.g. $\frac{d}{dx}[\ln(\sin x)] = \frac{\cos x}{\sin x} = \cot x$

$$\frac{d}{dx}[\ln(\cos x)] = \frac{-\sin x}{\cos x} = -\tan x$$

③ Derivatives of exponential functions and log functions with base different from e.

E.g. $\boxed{y = 2^x}$; $y = \log_2 x$

Goal: Find $\frac{dy}{dx}$?

$$\frac{d}{dx}[2^x]$$

$$= 2^x \cdot \ln(2)$$

Take the ln of both sides:

$$\ln(y) = \ln(2^{\boxed{x}})$$

$$\ln(y) = x \ln(2)$$

$$\frac{dy}{dx} = y \ln(2) = \boxed{2^x \cdot \ln 2}$$

$$\longrightarrow \frac{d}{dx}[\ln(y)] = \frac{d}{dx}[x \cdot \ln(2)]$$

$$\frac{1}{y}\boxed{\frac{dy}{dx}} = \ln(2)$$

$$\frac{d}{dx}[3^x] = 3^x \cdot \ln(3)$$

$$\frac{d}{dx}[\pi^x] = \pi^x \cdot \ln(\pi)$$

In general, $\boxed{\frac{d}{dx}[a^x] = a^x \cdot \ln(a)}$

(a is any base)

$$\boxed{\frac{d}{dx}[a^u] = a^u \cdot \ln(a) \cdot \frac{du}{dx}}$$

What about the log function?

$y = \log_2 x \longrightarrow$ find $\frac{dy}{dx}$?

$\longleftrightarrow 2^y = x \longrightarrow \frac{d}{dx}[2^y] = \frac{d}{dx}[x]$

$$2^y \cdot \ln(2) \cdot \boxed{\frac{dy}{dx}} = 1$$

$$\frac{dy}{dx} = \frac{1}{2^y \cdot \ln(2)} = \boxed{\frac{1}{x \ln 2}}$$