$$\frac{d}{dx}\left[\log(x)\right] = \frac{1}{x\ln(10)}$$

$$\frac{d}{dx}\left[\log_{2018}(x)\right] = \frac{1}{x\ln(2018)}$$

In general,

$$\boxed{\frac{d}{dx}\left[\log_a(x)\right] = \frac{1}{x\ln(a)}}$$

(a is any base)

$$\boxed{\frac{d}{dx}\left[\log_a(u)\right] = \frac{u'}{u\ln(a)}}$$

To sum up,

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[e^u\right] = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\ln x\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[\ln u\right] = \frac{u'}{u}$$

More generally,

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln(a)$$

$$\frac{d}{dx}\left[a^u\right] = a^u \cdot \ln(a) \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u\ln(a)}$$

Method of Logarithmic Differentiation

Useful Properties of the natural log function.

1. $\ln[uv] = \ln(u) + \ln(v)$
2. $\ln\left[\frac{u}{v}\right] = \ln(u) - \ln(v)$

③ $\ln[u^p] = p\ln(u)$

E.g. to illustrate logarithmic differentiation:

$\boxed{y = x^x}$. Find $\frac{dy}{dx}$.

Step 1: $\ln y = \ln(x^x)$ (take ln of both sides)

$\ln y = x\ln(x)$ (The 3^{rd} property of natural log)

Step 2: Differentiate both sides w.r.t. x.

$$\frac{d}{dx}[\ln y] = \frac{d}{dx}[x \cdot \ln(x)]$$ product rule

rule for ln & chain rule

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 \cdot \ln(x) + x \cdot \frac{1}{x}$$

algebra

$$\frac{1}{y} \cdot \boxed{\frac{dy}{dx}} = \ln(x) + 1$$

x^x

$$\frac{dy}{dx} = \boxed{y}(\ln(x) + 1) = \boxed{x^x(\ln(x) + 1)}$$

Step 3: Conclusion: $\frac{d}{dx}\left[x^x\right] = x^x\left(\ln x + 1\right)$

Ex. Find the derivative w.r.t. x of

$$y = (\ln x)^{\cos x}$$

$$y = (\ln x)^{\cos x}$$

$$\ln y = \ln\left((\ln x)^{\cos x}\right)$$

$$\ln y = \underbrace{\cos x}_{f} \cdot \underbrace{\ln(\ln x)}_{g}$$

product rule

$$\frac{1}{y}\cdot\boxed{\frac{dy}{dx}} = \left(-\sin x \cdot \ln(\ln x) + \cos x \cdot \frac{1}{\ln x}\cdot\frac{1}{x}\right)$$

$$\boxed{(\text{Stuff})\cdot(\ln x)^{\cos x}}$$

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Other examples where logarithmic differentiation method works well.

E.g. $\frac{d}{dx}\left[\frac{(2x+5)^{10}\cdot(3x-1)^7}{\sqrt{x^2+17}}\right]$

$$y = \boxed{\frac{(2x+5)^{10}\cdot(3x-1)^7}{\sqrt{x^2+17}}}$$

$$\ln y = \ln(2x+5)^{10} + \ln(3x-1)^7 - \ln\left(\sqrt{x^2+17}\right)$$

$$\ln y = 10\ln(2x+5) + 7\ln(3x-1) - \frac{1}{2}\ln(x^2+17)$$

$$\frac{1}{y}\frac{dy}{dx} = \underbrace{\left(\frac{20}{2x+5} + \frac{21}{3x-1} - \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}x}{x^2+17}\right)}$$

$$\frac{dy}{dx} = (\text{Stuff})\cdot\boxed{y}$$