4.10. Antiderivative.

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Goal: Understand the concept of antiderivative and basic antiderivative formulas

Definition of an antiderivative of a function:

f: function defined on an interval I.

An antiderivative of f is a function F such that

$$F'(x) = f(x) \text{ for every } x \text{ in } I.$$

In short, an antiderivative of a function is another function whose derivative is equal to the given function.

E.g. $f(x) = x$ defined on $(-\infty, \infty)$

Find an antiderivative of this function.

An answer is $F(x) = \frac{x^2}{2}$

$F(x) = \frac{x^2}{2} + 100$ is another antiderivative of f.

In general, $F(x) = \frac{x^2}{2} + C$ is an antiderivative of f.

E.g. $f(x) = x^2$.

$F(x) = \frac{x^3}{3}$ is an antiderivative of f.

(Reason: $F'(x) = \left(\frac{x^3}{3}\right)' = \frac{3x^2}{3} = x^2$)

In general, any antiderivative of $f(x) = x^2$ must have the form:

$$F(x) = \boxed{\frac{x^3}{3} + C},$$ where C is any constant

The formula $\frac{x^3}{3} + C$ is called the most general antiderivative of the function $f(x) = x^2$.

E.g. $f(x) = x^{2018}$

Q: Find the most general antiderivative of f?

$$\boxed{\frac{x^{2019}}{2019} + C}$$,

(Reason: $\left(\frac{x^{2019}}{2019} + C\right)' = \frac{\cancel{2019}\, x^{2018}}{\cancel{2019}} + 0$

$= x^{2018}$)

Very Important Notation:

$$\boxed{\int f(x)\,dx = \text{the most general antiderivative of } f(x)}$$

Basically, $\int f(x)\,dx = F(x) + C$

where $F(x)$ is such that $F'(x) = f(x)$ and C is a constant.

E.g. $\displaystyle\int x^3\,dx = \frac{x^4}{4} + C$

In general, $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

(Note: $n \neq -1$)

$$\int \frac{1}{x}\,dx \overset{?}{=} \ln|x| + C$$

$$\int \sqrt{x}\,dx = \int x^{\overset{n}{\boxed{1/2}}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \boxed{\frac{2}{3}x^{\frac{3}{2}} + C}$$

Table of very useful antiderivatives

Function $f(x)$	Antiderivative : $\int f(x)\,dx$
$f(x) = x^n, \; n \neq -1$	$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \;;\; n \neq -1$
$f(x) = \dfrac{1}{x}$	$\int \dfrac{1}{x}\,dx = \ln\lvert x\rvert + C$
$f(x) = \sin x$	$\int \sin x\,dx = -\cos(x) + C$
$f(x) = \cos x$	$\int \cos x\,dx = \sin(x) + C$
$f(x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan(x) + C$
$f(x) = \sec(x)\tan(x)$	$\int \sec(x)\tan(x)\,dx = \sec(x) + C$

$f(x) = \csc^2(x)$	$\int \csc^2(x)\,dx = -\cot(x) + C$
$f(x) = \csc(x)\cot(x)$	$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$
$f(x) = e^x$	$\int e^x\,dx = e^x + C$
$f(x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$
$f(x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$
$f(x) = k$; k is a constant	$\int k\,dx = kx + C$