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Useful Properties of the antiderivative

Linearity:

$$\boxed{\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx}$$

$$\boxed{\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx \text{ ; } k \text{ is a constant}}$$

(We can pull a constant out of the antiderivative)

Ex: Find the given antiderivative using the formulas in the table.

1. $\int \left(7x^{2/5} + 8x^{-4/5}\right) dx$

2. $\int \left(2\sin(x) - \sec^2(x)\right) dx$

3. $\int e^2\,dx = e^2 x + C$

(Answer $\boxed{\text{not}}$ $\boxed{e^2 + C}$)

4. $\int \left(4\sqrt{x} - \sqrt[4]{x}\right) dx$

⑤ $\int (x+1)(2x-1)\,dx$

⑦ $\int \frac{2+x^2}{1+x^2}\,dx$

⑥ $\int \frac{2x^5-\sqrt{x}}{x}\,dx$

Linearity

① $\int \left(7x^{2/5} + 8x^{-4/5}\right)dx = 7\int x^{2/5}\,dx + 8\int x^{-4/5}\,dx$

$= 7 \cdot \frac{x^{\frac{2}{5}+1}}{\frac{2}{5}+1} + 8 \cdot \frac{x^{-\frac{4}{5}+1}}{-\frac{4}{5}+1} + C$ (anti-power rule)

$= 7 \cdot \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + 8 \cdot \frac{x^{\frac{1}{5}}}{\frac{1}{5}} + C$

$= \boxed{5x^{7/5} + 40x^{1/5} + C}$

Simplify.

(2) $\int (2\sin x - \sec^2 x)\, dx = -2\cos(x) - \tan(x) + C$

(4) $\int (4\sqrt{x} - \sqrt[4]{x})\, dx = 4\int x^{1/2}\, dx - \int x^{1/4}\, dx$

Split by linearity and rewrite as power x^n.

$$= 4 \cdot \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + C$$

$$= \boxed{6 \cdot x^{3/2} - \frac{4}{5} x^{5/4} + C}$$

(5) $\int (x+1)(2x-1)\, dx = \int (2x^2 + x - 1)\, dx$

Distribute

$$= \boxed{\frac{2x^3}{3} + \frac{x^2}{2} - x + C}$$

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$$x^{1/2}$$

⑥ $$\int \frac{2x^5 - \sqrt{x}}{x}\,dx = \int\left(\frac{2x^5}{x} - \frac{\sqrt{x}}{x^1}\right)dx$$

$$= \int\left(2x^4 - x^{-1/2}\right)dx = \frac{2x^5}{5} - \frac{x^{1/2}}{1/2} + C$$

$$= \frac{2x^5}{5} - 2x^{1/2} + C.$$