4.3. Find Maxima and Minima of a function

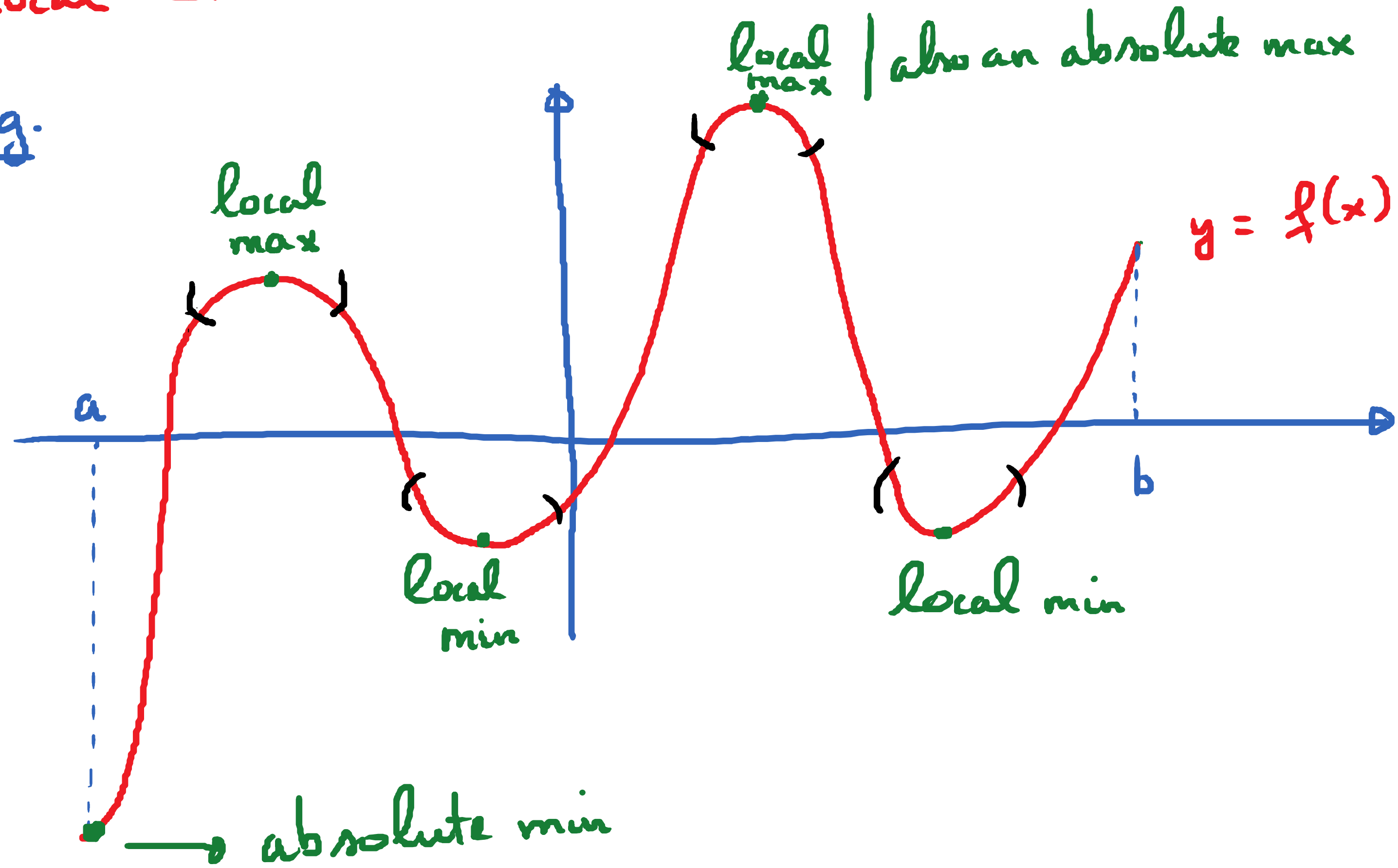
Monday, July 30, 2018 10:43 AM

Goals: (1) Find critical number(s) (critical points) of a function.

(2) Apply the closed interval method to find maxima and minima of a function on a closed interval $[a, b]$.

Local Extrema vs. Absolute Extrema.

Eg.



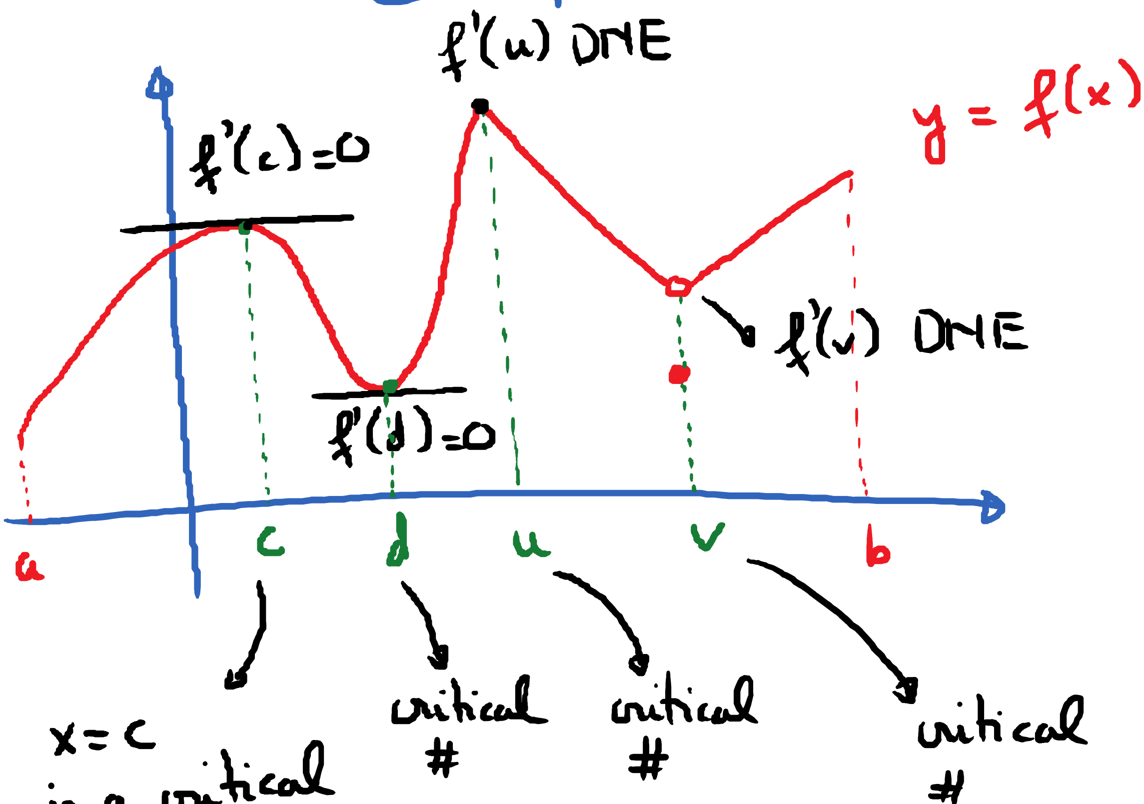
Tuesday, July 31, 2018 7:41 AM

Critical numbers (critical points) of a function.

Definition: Let c be a number in the domain of a function $y = f(x)$. We say that c is a critical number of f if

Either (1) $f'(c) = 0$

or (2) $f'(c)$ does not exist.



$x = c$ is a critical # of f b/c $f'(c) = 0$

Eg. $f(x) = \sqrt{x}$, Domain: $[0, \infty)$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$f'(0)$ DNE ; $x = 0$ is in the domain of f

$\longrightarrow$ $x = 0$ is a critical # of f.

$f'(-1)$ DNE ; $x = -1$ is NOT in the domain of f $\longrightarrow$ $x = -1$ is NOT a critical # of f.

Eg. $f(x) = (x+1)^2$. Domain: $(-\infty, \infty)$

$f'(x) = 2(x+1)$

$f'(-1) = 0$, $x = -1$ is in the domain of f.

$\longrightarrow$ $x = -1$ is a critical # of f.

How to find the critical numbers of a function f

1. Find the domain of f.
2. Find f'
3. Set $f'(x) = 0$
4. Find the values of x for which f' is undefined

Final Step: the values in 3 and 4 that are in the domain of f are the critical #s of f.

Ex. Find the critical number(s) of the given function.

ⓐ $f(x) = x^3 - 6x^2 + 9x + 1$

ⓑ $g(x) = \ln(1 - x)$

ⓒ $h(x) = \dfrac{4x}{1 + x^2}$

ⓓ $j(x) = x^{\frac{3}{5}}(4 - x)$

ⓔ $u(x) = 4\sqrt{x} - x^2$

Solution:

ⓐ * Domain of f: $(-\infty, \infty)$

* $f'(x) = 3x^2 - 12x + 9$

* $f'(x) = 0 \leftrightarrow 3x^2 - 12x + 9 = 0$

$$3(x^2 - 4x + 3) = 0$$

$$3(x - 1)(x - 3) = 0$$

$$\boxed{x = 1} \; ; \; \boxed{x = 3}$$

* Values of x for which $f'(x)$ is undefined: NONE.

* Critical #s of f are $\boxed{x = 1}$ and $\boxed{x = 3}$

ⓑ $g(x) = \ln(\boxed{1 - x})$

* Domain of g: $1 - x > 0 \leftrightarrow 1 > x$.

Domain: $(-\infty, 1)$

* $g'(x) = \dfrac{-1}{1-x}$
* $g'(x) = 0$; $\dfrac{-1}{1-x} = 0$: No solution.
* $g'(x)$ is undefined when $x = 1$
* $x = 1$ is <u>not</u> in the domain of g.

Conclusion: g has NO critical #.

(c) * Domain: $(-\infty,\infty)$

* $h'(x) = \dfrac{(1+x^2)\cdot 4 - 4x\cdot(2x)}{(1+x^2)^2} = \dfrac{4+4x^2-8x^2}{(1+x^2)^2}$

$h'(x) = \dfrac{4-4x^2}{(1+x^2)^2}$

* $h'(x)=0 \longleftrightarrow 4-4x^2=0$; $4x^2=4$; $x^2=1$

$x=\pm 1$

* $h'(x)$ undefined : None

Conclusion: critical #s are $x=1$ and $x=-1$

(d) $j(x) = x^{\frac{3}{5}}(4-x) = 4x^{\frac{3}{5}} - x^{\frac{8}{5}}$

* Domain : $(-\infty,\infty)$

* $j'(x) = 4\cdot\frac{3}{5}\cdot x^{-2/5} - \frac{8}{5}x^{3/5}$

$= \dfrac{12}{5x^{2/5}} - \dfrac{8x^{3/5}}{5}\cdot\dfrac{x^{2/5}}{x^{2/5}} = \dfrac{12-8x}{5x^{2/5}}$

* $j'(x)=0 \longleftrightarrow 12-8x=0 \longleftrightarrow x=\frac{3}{2}$

* $j'(x)$ is undefined when $5x^{2/5}=0 \longleftrightarrow x^{2/5}=0$

$\longleftrightarrow x=0$

* Conclusion: $x=\frac{3}{2}$; $x=0$ are the critical #s.