Tuesday, July 31, 2018 8:40 AM

(e) $u(x) = 4\sqrt{x} - x^2$

* Domain : $[0, \infty)$

* $u'(x) = \dfrac{2}{\sqrt{x}} - 2x = \dfrac{2 - 2x\sqrt{x}}{\sqrt{x}} = \dfrac{2 - 2x^{3/2}}{x^{1/2}}$

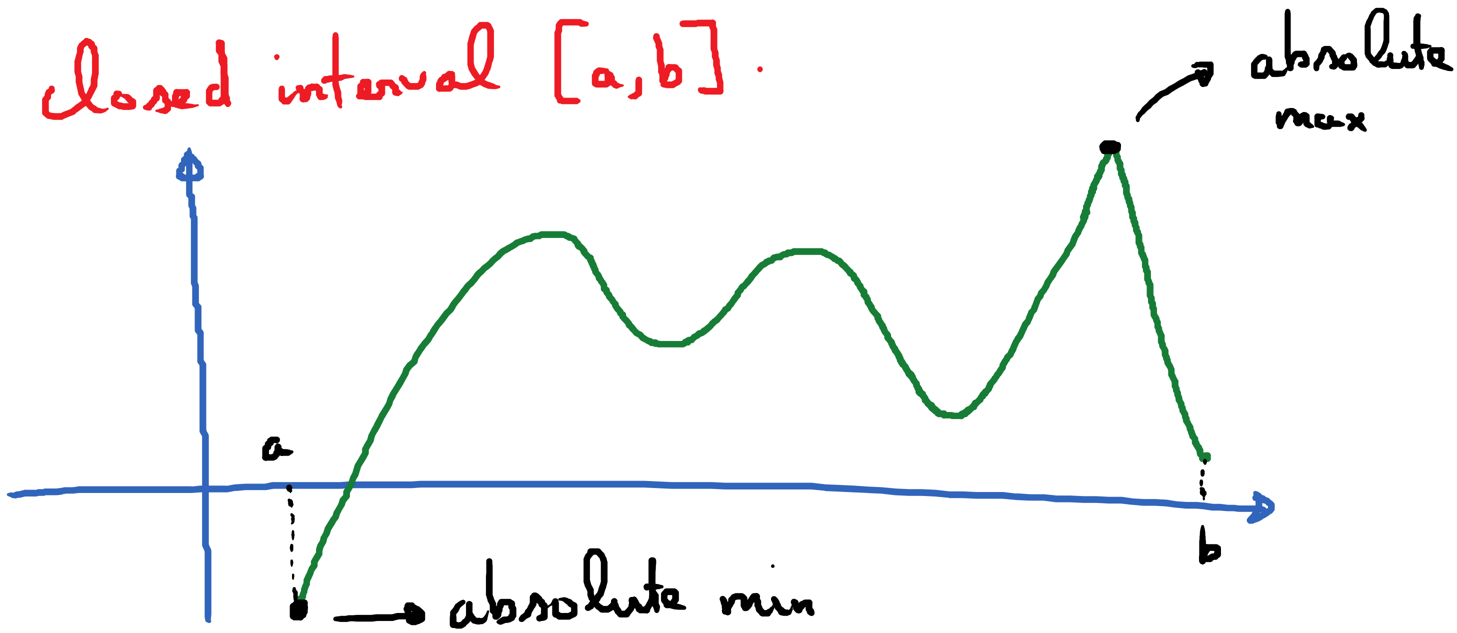
* $u'(x) = 0 \Leftrightarrow 2 - 2x^{3/2} = 0 \Leftrightarrow x^{3/2} = 1$

$\Leftrightarrow x = 1$

* $u'(x)$ is undefined when $x^{1/2} = 0 \Leftrightarrow x = 0$

* Conclusion: critical #s are $x = 1$ and $x = 0$.

* Closed Interval Method to find the absolute max and absolute min of a function on a closed interval $[a, b]$.



Point: An absolute max or absolute min of a function f occurs either at a critical number of f or at an endpoint of the interval.

E.g. $f(x) = x^3 - 6x^2 + 9x + 1$; over $[-1, 2]$

Find absolute max and absolute min of f on $[-1, 2]$.

Step 1: Find all the critical #s of f within $[-1, 2]$

Domain: $[-1, 2]$

$f'(x) = 3x^2 - 12x + 9$.

$f'(x) = 0$ when $x = 1$; $x = 3$

$f'(x)$ undefined: None.

critical #'s in $[-1, 2]$: $\boxed{x = 1}$

Step 2: Find $f(-1)$; $f(2)$; $f(1)$

$f(-1) = -15$

$f(2) = 3$

$f(1) = 5$

Step 3: Absolute max value of f is 5 and it occurs at $x = 1$.

Coordinates of the absolute max: $(1, 5)$

Absolute min value of f is -15 and it occurs at $x = -1$.

Absolute min $(-1, -15)$

Summary of the closed interval method to find abs. max / min of f on $[a,b]$

(1) Find all the critical #s of f within $[a,b]$

(2) Evaluate f at the critical #s in (1)
and Evaluate f at the endpoints a and b.

(3) The largest value in (2) gives the abs. max
The smallest value in (2) gives the abs. min