4.5. How does the derivative affect the shape of a graph?

Thursday, August 2, 2018 7:33 AM

Goals:

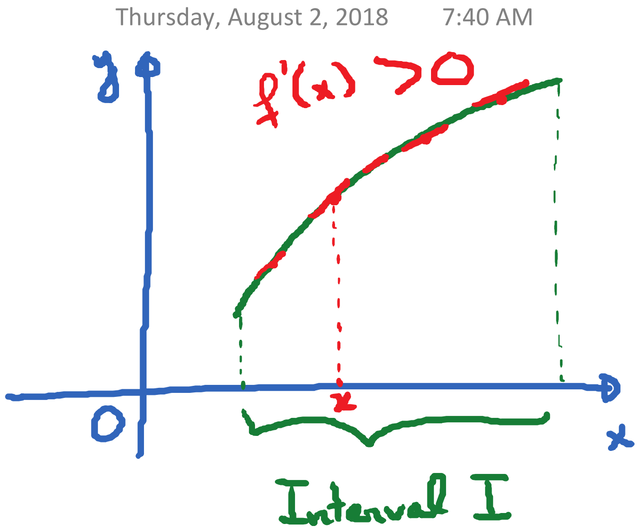
1. Use the first derivative to determine the intervals of increasing/decreasing of a function.
2. Find local extrema of a function by the first derivative test.
3. Use the second derivative to determine the intervals of concavity of a function.
4. Use second derivative test to determine local extrema.

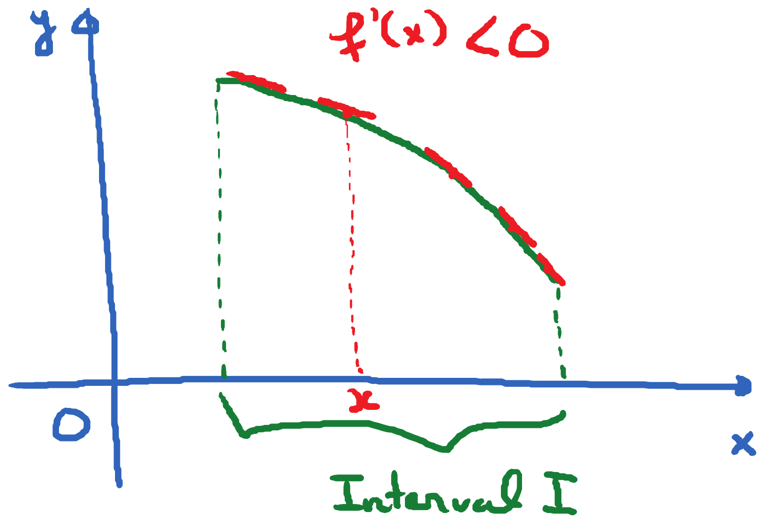
1 What does f' tell us about the graph of f?

Theorem: (I) If $f'(x) > 0$ for every x in an interval I, then f is increasing on I.

(II) If $f'(x) < 0$ for every x in an interval I, then f is decreasing on I.

Proof: Need M.V.T.





(I) $f' > 0 \to \uparrow$ (II) $f' < 0 \Rightarrow \downarrow$

E.g. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$. $D = (-\infty, \infty)$

Q: Determine the interval(s) on which f is increasing and on which f is decreasing

$f'(x) = 12x^3 - 12x^2 - 24x$

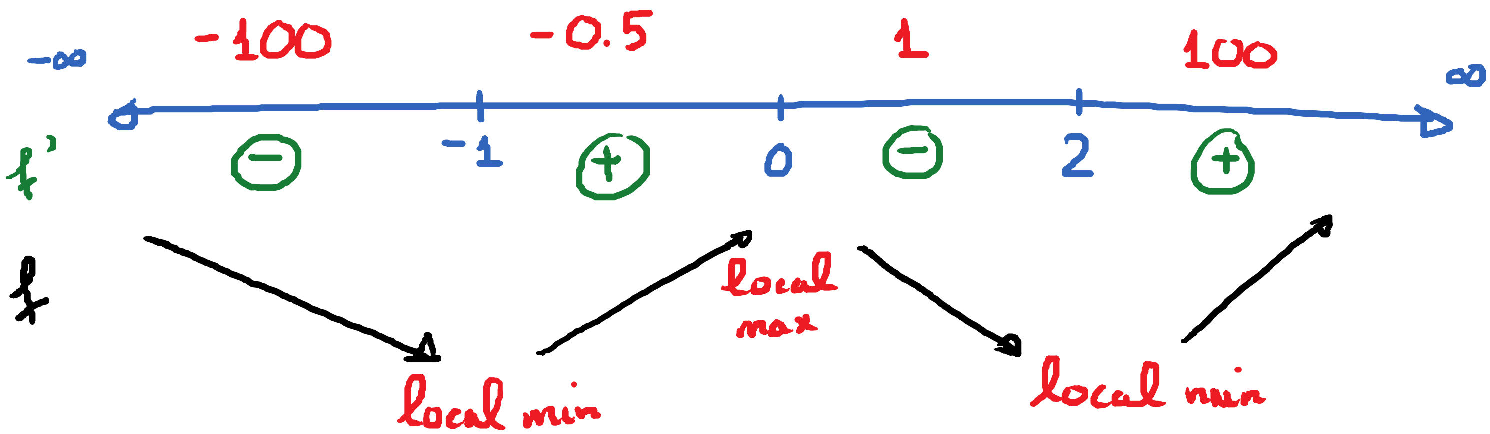
Step 1: Find critical #s:

$f'(x) = 0 \leftrightarrow 12x^3 - 12x^2 - 24x = 0$

$\leftrightarrow 12x(x^2 - x - 2) = 0 \leftrightarrow 12x(x-2)(x+1) = 0$

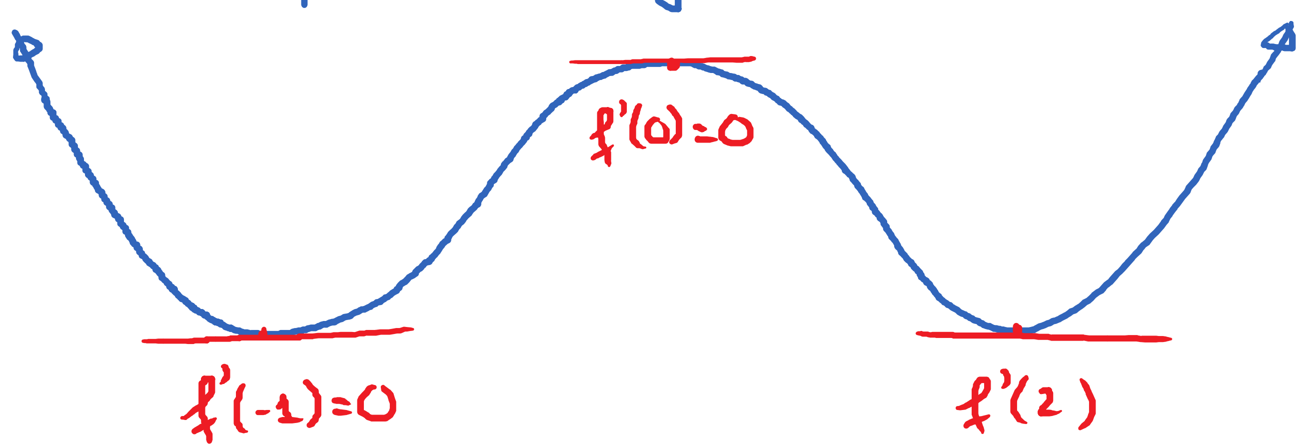
$\leftrightarrow x = 0;\ x = 2;\ x = -1$

Step 2: Draw number line and use test points



Conclusion: f is decreasing on $(-\infty,-1) \cup (0,2)$

f is increasing on $(-1,0) \cup (2,\infty)$



f has a local min @ $x=-1$ and @ $x=2$

y-coord $= f(-1)$ y-coord $= f(2)$

f has a local max @ $x=0$, y-coord $= f(0)$

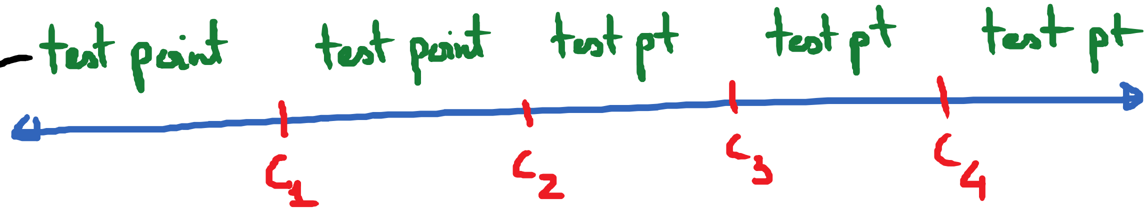
First Derivative Test to find local min / local max

Step 1: Find f'

Step 2: Find the critical #s

- $f' = 0 \rightarrow$ solve for x
- f' undefined $\rightarrow$ solve for x

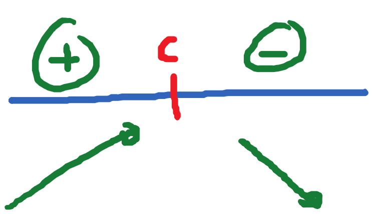
Step 3: Draw a number line and use test points.



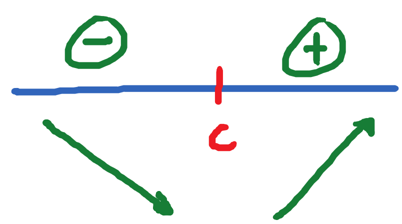
plug test point into f' to determine sign of f' on the interval.

critical #

Step 4:



f' changes from ⊕ to ⊖ across $c \rightarrow f$ has a local max @ $x = c$



f' changes from ⊖ to ⊕ across $c \rightarrow f$ has a local min @ $x = c$.

Note: If f' does not change sign at c, then f has neither a local max nor a local min at $x=c$

E.g. $f(x) = 2\ln(x) - 5\arctan(x)$

Use the first derivative test to find the local min and local max of f.

$$f'(x) = \frac{2}{x} - \frac{5}{1+x^2}$$

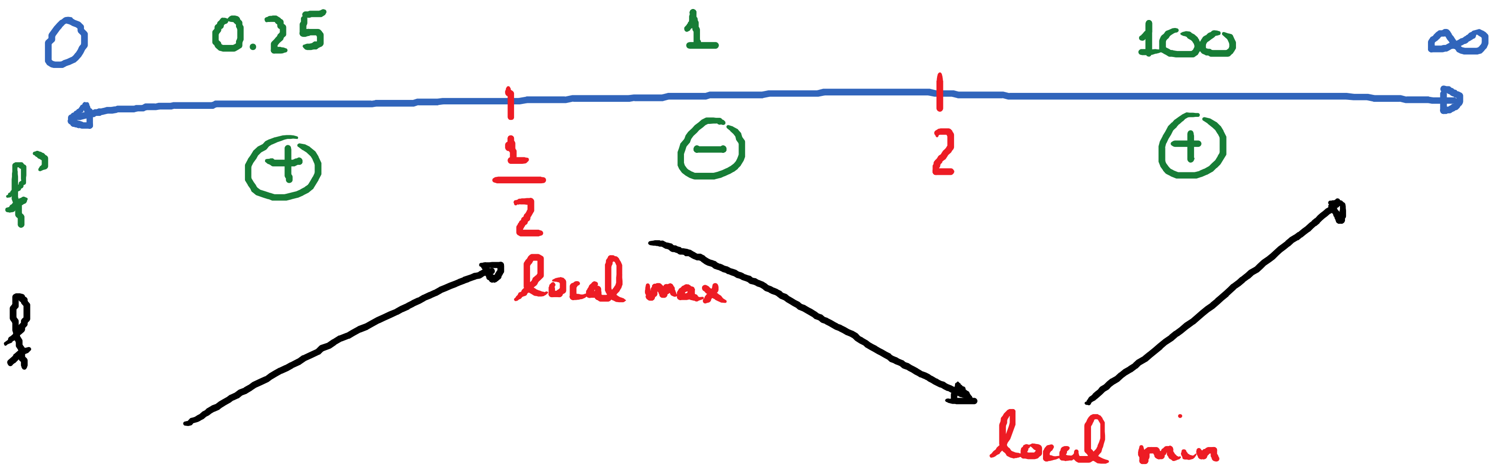
$$= \frac{2(1+x^2) - 5x}{x(1+x^2)}$$

$$f'(x) = \frac{2x^2 - 5x + 2}{x(1+x^2)} = \frac{(2x-1)(x-2)}{x(1+x^2)}$$

Find critical #s: $(2x-1)(x-2) = 0$

$\longrightarrow x = \frac{1}{2}$; $x = 2$. $(f' = 0)$

$x(1+x^2) = 0 \longrightarrow x = 0$ (f' is undefined)



f has a local max @ $x = \frac{1}{2}$:

$$f\left(\tfrac{1}{2}\right) = 2\ln\left(\tfrac{1}{2}\right) - 5\arctan\left(\tfrac{1}{2}\right) \approx -3.704$$

Local max $\left(\frac{1}{2}, -3.704\right)$

f has a local min @ $x = 2$

$$f(2) = 2\ln(2) - 5\arctan(2) \approx -4.149$$

Local min $(2, -4.149)$