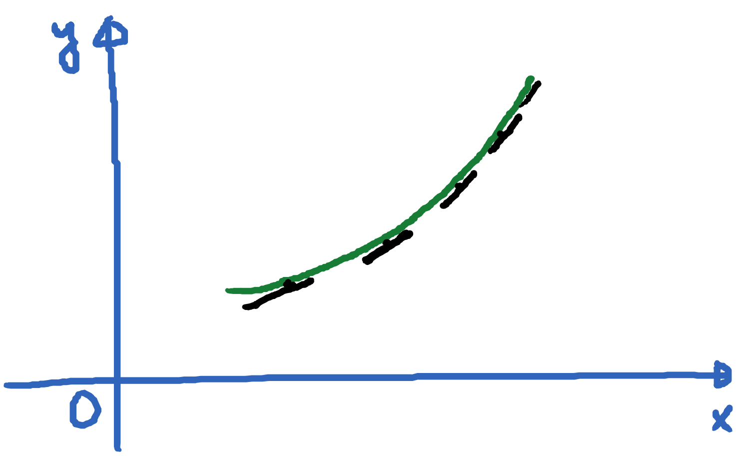
2 What does f'' tell us about the graph of f?

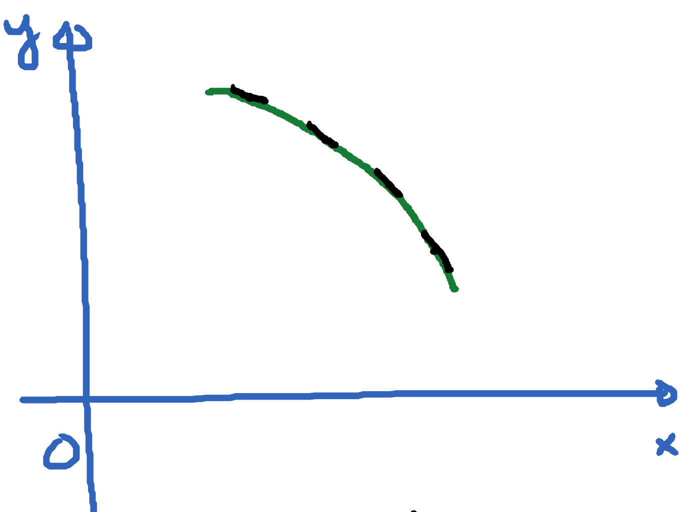
Theorem: I. If $f''(x) > 0$ for every x in an interval I, then f is concave up on I.

positive

II. If $f''(x) < 0$ for every x in an interval I, then f is concave down on I.

negative





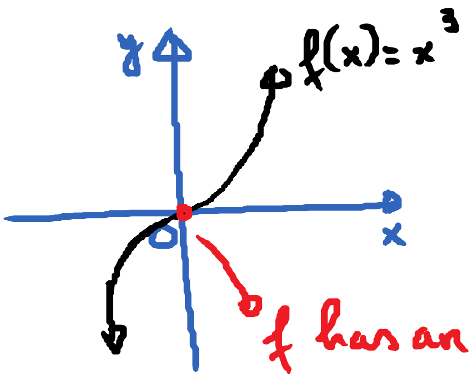
$f' \uparrow \leftrightarrow f'' > 0$ $\qquad$ $f' \downarrow \leftrightarrow f'' < 0$

I. Concave up $\leftrightarrow f'' > 0$ $\qquad$ II. Concave down $\leftrightarrow f'' < 0$

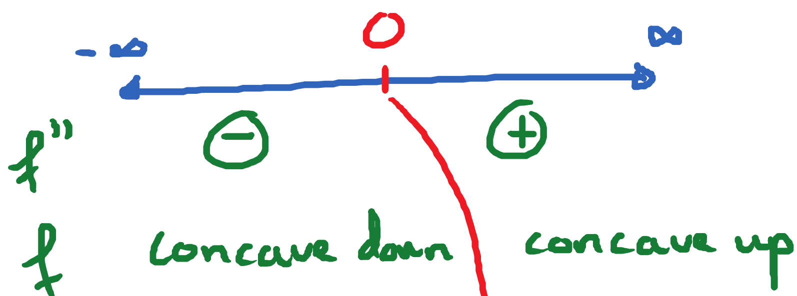
Definition: Inflection Point.

If $f''(c) = 0$ or $f''(c)$ is undefined and f'' changes its sign across $x = c$, then f has an inflection point @ $x = c$.

E.g. $f(x) = x^3$; $f'(x) = 3x^2$; $f''(x) = 6x$



f has an inflection point at $x = 0$



inflection point.

E.g. $g(x) = 6xe^{-x^2}$

Determine interval(s) of concavity.

Find inflection points

$$g'(x) = 6\cdot\left(e^{-x^2} + x\cdot e^{-x^2}\cdot(-2x)\right)$$

$$= 6\left(e^{-x^2} - 2x^2\cdot e^{-x^2}\right)$$

$$g'(x) = 6e^{-x^2}\left(1-2x^2\right)$$

$$g''(x) = 6\cdot\left[e^{-x^2}\cdot(-4x) + (1-2x^2)\cdot e^{-x^2}\cdot(-2x)\right]$$

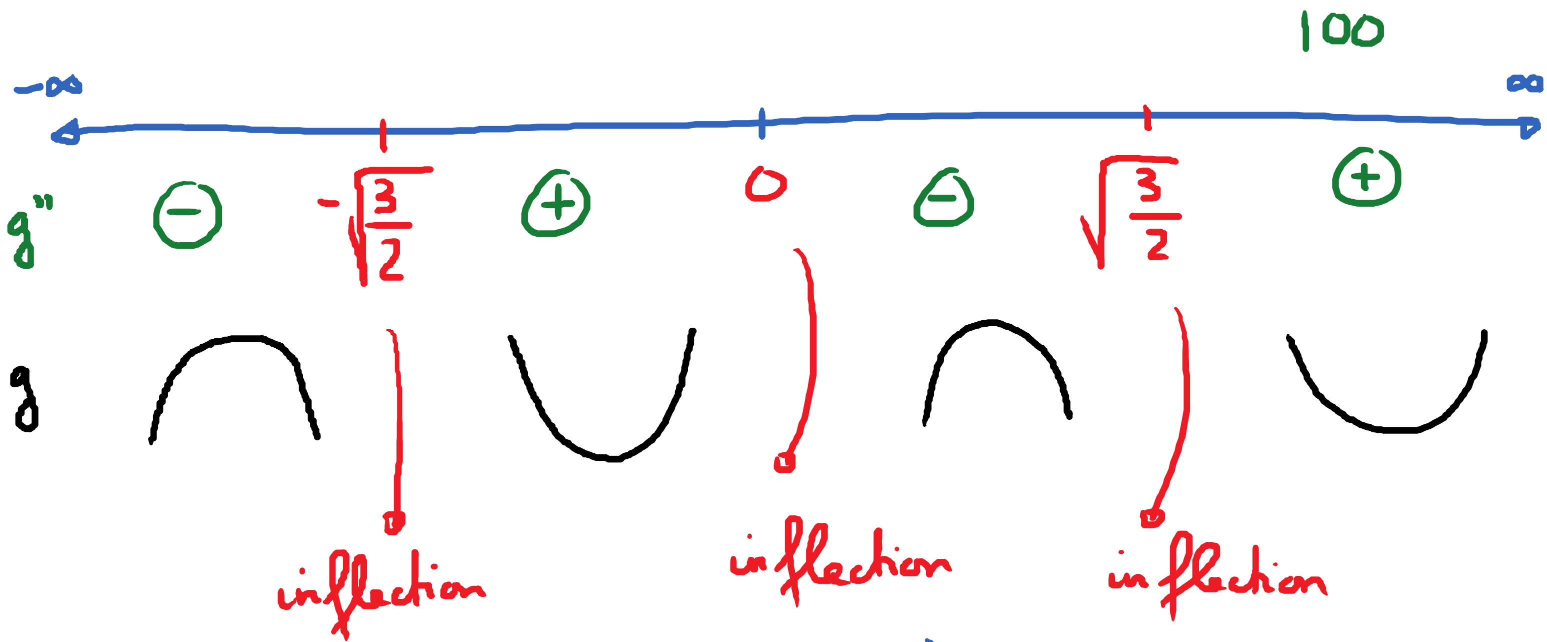
$$= 6\cdot\left[-2xe^{-x^2}\left(2+1-2x^2\right)\right]$$

$$g''(x) = -12\boxed{x}\boxed{e^{-x^2}}\left(\boxed{3-2x^2}\right)$$

$$g''(x) = 0 \longleftrightarrow \boxed{x=0} \text{ or } 3-2x^2=0$$

$$2x^2 = 3$$

$$x^2 = \frac{3}{2}; \quad \boxed{x = \pm\sqrt{\frac{3}{2}}}$$



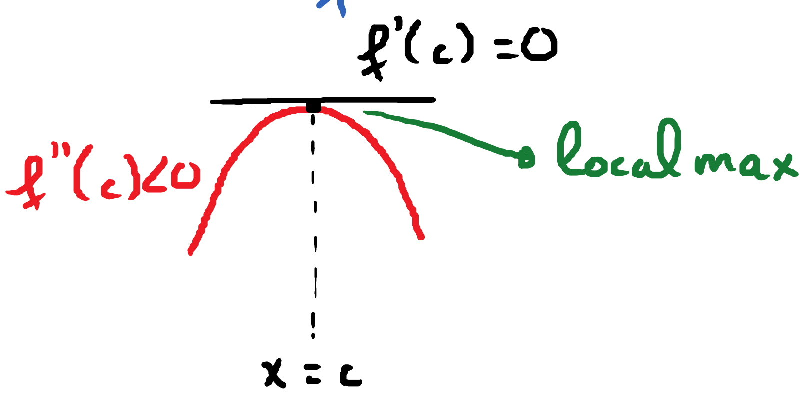
g is concave down on $\left(-\infty, -\sqrt{\frac{3}{2}}\right) \cup \left(0, \sqrt{\frac{3}{2}}\right)$

g is concave up on $\left(-\sqrt{\frac{3}{2}}, 0\right) \cup \left(\sqrt{\frac{3}{2}}, \infty\right)$

Note: The second derivative can also be used in many cases to determine local min/max.

* $f'(c) = 0$ and $f''(c) < 0$

$\Longrightarrow$ f has a local max at $x = c$.

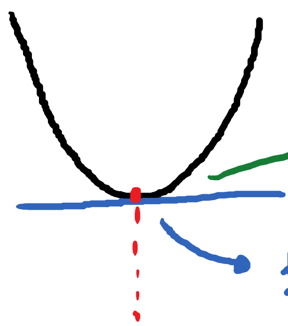


Thursday, August 2, 2018 9:02 AM

$f'(c) = 0$; $f''(c) > 0$.

$\Longrightarrow$ f has a local min at $x = c$

$f''(c) > 0$



local min @ c.

$f'(c) = 0$

$x = c$

$\longrightarrow$ This is called the second derivative test.