4.7. Optimization Problems

Monday, August 6, 2018 7:37 AM

Goal: Solve optimization problems.

Key: Either maximize or minimize some quantity.

Find a function $f(x)$ to describe that quantity.

Case 1: x belongs to a closed interval $[a,b]$

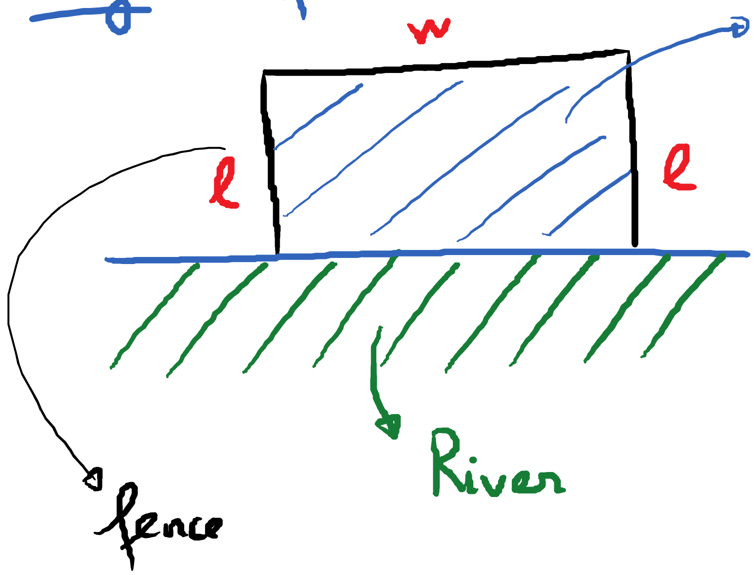
→ Closed interval method
- Find critical #s of f in (a,b)
- Find values of f at those critical #s and at endpoints

→ compare them

Case 2: x belongs to an open interval, E.g., $(-\infty,\infty)$

→ First derivative test.

E.g. A farmer has 2400 ft of fencing.



Rectangular piece of land.

Find the dimensions of the rectangle that will enclose the piece of land with the largest area.

Constraint: $2l + w = 2400$

Goal: Maximize the area $A = l \cdot w$ with respect to this constraint.

Turn A into a function of one variable by solving for a variable in terms of the other one in the constraint equation and plug in the formula for A.

$$2l + w = 2400 \longrightarrow w = 2400 - 2l.$$

Plug in the formula for A:

$$A = \ell \cdot (2400 - 2\ell)$$

$\rightarrow$ $A(\ell) = 2400\ell - 2\ell^2$.

Restriction on ℓ: ℓ belongs to $[0, 2400]$

$\rightarrow$ The problem becomes:

Maximize the function $A(\ell) = 2400\ell - 2\ell^2$ on the closed interval $[0, 2400]$

$\longrightarrow$ Closed Interval Method.

Step 1: $A'(\ell) = 2400 - 4\ell$

$A'(\ell) = 0 \leftrightarrow 2400 - 4\ell = 0$

$\leftrightarrow \ell = 600 \rightarrow$ critical #.

Step 2: $A(0) = 0$

$A(2400) = 0$ } Plug endpoints into $A(\ell)$

$A(600) = 2400 \cdot 600 - 2 \cdot (600)^2 = \boxed{720000}$ ← max

Area is maximum when $l = 600$ ft, $w = 1200$.

The maximum area is 720000 ft^2

Strategy for solving optimization problems.

(1) Understand the problem < given / want

(2) Draw a diagram.

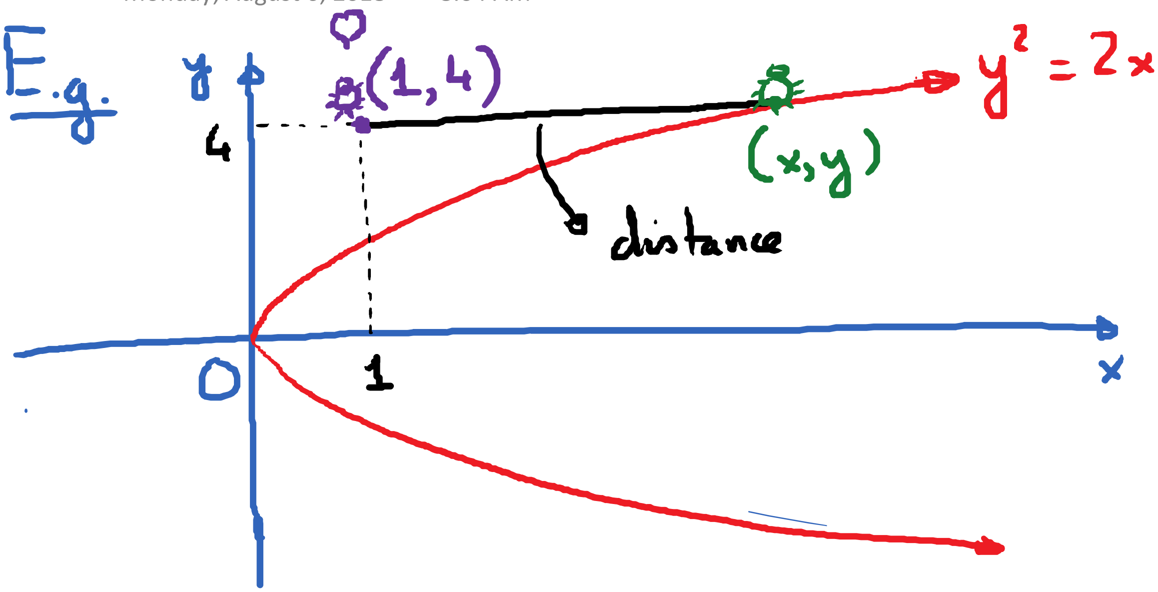
(3) Introduce notations <
- Quantity to maximize or minimize, say, Q
- Unknowns (quantities related to Q)

(4) Find an equation that gives Q in terms of the unknowns.

(5) Turn Q into a function of one variable by using the constraint(s) on the unknowns.

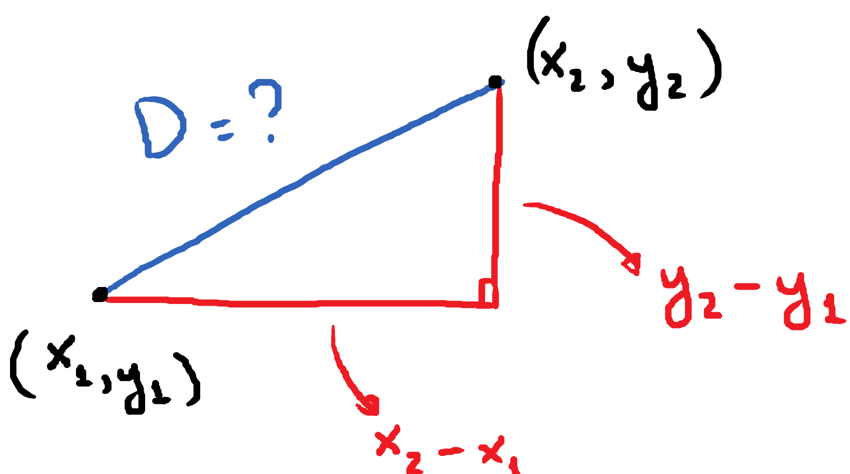
(6) Use the closed interval method or the first derivative test to find max or min of Q.

E.g.



Find the point (x, y) on the parabola that is closest to the given point (1, 4).

<u>Goal</u>: Find (x, y) so that the distance from (x, y) to (1, 4) is the smallest (Find minimum)



distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$