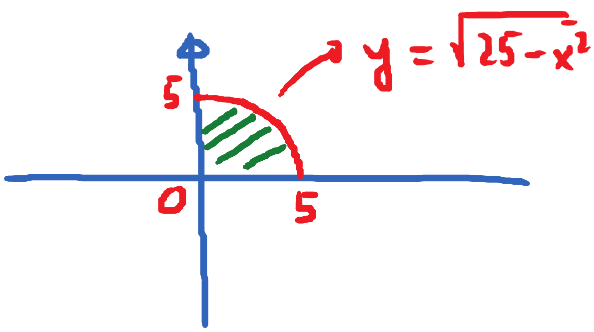
semicircle

5) $\int_0^5 \sqrt{25-x^2}\,dx$ = area under curve $y=\sqrt{25-x^2}$ over $[0,5]$



$$= \boxed{\frac{\pi \cdot (\text{Radius})^2}{4}} = \frac{25\pi}{4}$$

area formula for $\frac{1}{4}$ of a circle.

Fundamental Theorem of Calculus, Part I

→ Lower bound and upper bound of the integral will be variables!

E.g. $f(t)=t^2$; on $[1,3]$

Consider the integral:

FTC, part II

$$\int_1^x f(t)\,dt = \int_1^x t^2\,dt = \frac{t^3}{3}\Big|_1^x$$

the answer is

$x^3 \qquad 1$

$$= \boxed{\frac{x^3}{3} - \frac{1}{3}}$$

the answer is a function of x.

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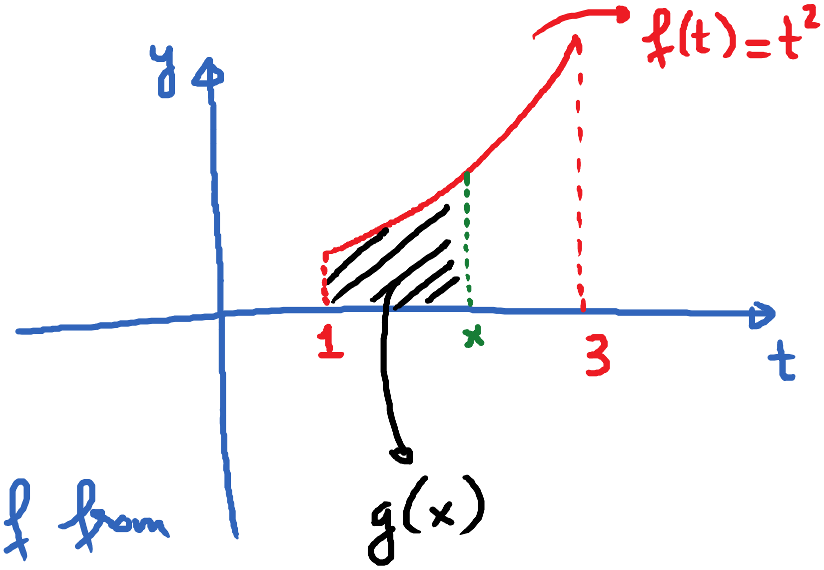
$$g(x) = \int_1^x f(t)\,dt$$



$g(1.5)$ = area under f from 1 to 1.5

$g(1.75)$ = area under f from 1 to 1.75

$g(2)$ = ________________ 1 — 2 .

$$\frac{d}{dx}\left(\int_1^x f(t)\,dt\right) = \frac{d}{dx}\left(\frac{x^3}{3} - \frac{1}{3}\right) = \boxed{x^2}$$

$f(t) = t^2$ $\qquad$ $f(x)$

Statement of FTC, part I.

$$\boxed{\boxed{\frac{d}{dx}}\left[\int_a^x \boxed{f(t)}\,dt\right] = \boxed{f(x)}}$$

take der. w.r.t. x of that function

integrand as a function of x

function x

In short, differentiation and integration are inverse operations of each other.

E.g. $\underbrace{\frac{d}{dx}}_{\substack{\text{take deri.}\\ \text{w.r.t. of } x}}\left(\underbrace{\boxed{\int_0^x \boxed{e^{t^2}}\, dt}}_{\text{function of } x}\right) = e^{x^2}$

F.T.C, part I

$$\frac{d}{dr}\left(\int_4^r \frac{d\theta}{\sqrt{16-\theta^2}}\right) = \frac{1}{\sqrt{16-r^2}}$$

F.T.C., part I

$$\frac{d}{dx}\left(\int_0^{\boxed{x}} \sec(t)\, dt\right) = \sec(x)$$

$$\frac{d}{dx}\left(\int_0^{\boxed{x^2}} \sec(t)\, dt\right) = \sec(x^2)\cdot 2x$$

by Chain Rule

More general formula for FTC, part I:

$$\frac{d}{dx}\left(\int_a^{\boxed{g(x)}} f(t)\,dt\right) = f(g(x)) \cdot \boxed{g'(x)}$$

differentiate g

chain rule

E.g. $$\frac{d}{dx}\left(\int_1^{\boxed{x^4}} \frac{1}{t^3+1}\,dt\right) = \boxed{\frac{1}{(x^4)^3+1}} \cdot \boxed{4x^3}$$

diff.

$$= \frac{4x^3}{x^{12}+1}$$

Ex.1. Find the given derivative:

(a) $$\frac{d}{dx}\left(\int_0^{\tan x} \sqrt{t+\sqrt{t}}\,dt\right) = \sqrt{\tan x + \sqrt{\tan x}} \cdot \sec^2 x$$

(b) $$\frac{d}{dx}\left(\int_{2x}^{\boxed{3x}} \frac{u^2-1}{u^2+1}\,du\right)$$

(c) $$\frac{d}{dx}\left(\int_{\sqrt{x}}^{2x} \arctan(t)\,dt\right)$$

(2) $$\frac{(3x)^2-1}{(3x)^2+1}\cdot 3 - \frac{(2x)^2-1}{(2x)^2+1}\cdot 2$$

$$= 3\cdot\frac{9x^2-1}{9x^2+1} - \frac{4x^2-1}{4x^2+1}\cdot 2.$$

(3) $$2\arctan(2x) - \frac{\arctan(\sqrt{x})}{2\sqrt{x}}$$

Ex. $$g(x) = \int_0^x (1-t^2)e^{t^2}\,dt$$

Q: On what interval is the function g decreasing?

$$g'(x) = \frac{d}{dx}\left(\int_0^x (1-t^2)e^{t^2}\,dt\right)$$

$$= \boxed{(1-x^2)e^{x^2}}$$

FTC, part I

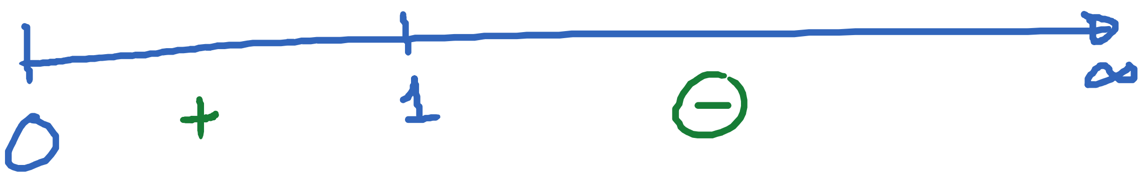
$$g'(x) = 0 \longleftrightarrow (1 - x^2)\, e^{x^2} = 0$$

$$\longleftrightarrow \cancel{e^{x^2} = 0} \quad ; \quad 1 - x^2 = 0$$

$$(\text{b/c } e^{x^2} > 0) \qquad x^2 = 1 \longleftrightarrow x = \pm 1$$

Since the lower bound is 0, the critical # is $x = 1$.

Test pt 0.5 Test pt 100



g is decreasing on $(1, \infty)$