## 5.5. Integration by Substitution on u-sub Thursday, August 9, 2918 9:59 AM

Recall: 
$$\int x^{2} dx = \frac{x^{3}}{3} + C ; \int y^{2} dy = \frac{y^{3}}{3} + C$$
$$\int u^{2} du = \frac{u^{3}}{3} + C ; \int 3^{2} dy = \frac{3^{3}}{3} + C$$

E.g. of 
$$u - sub$$

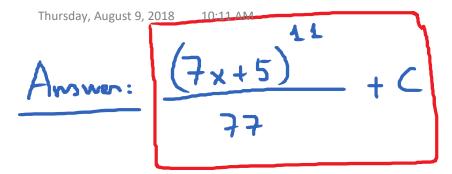
$$\frac{du}{dx} = 7 \rightarrow du = 7 dx$$

$$\frac{du}{dx} = \frac{10}{4} dx$$

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$$\int_{u}^{40} \frac{du}{7} = \frac{1}{7} \int_{u}^{40} du = \frac{1}{7} \cdot \frac{u^{11}}{11} + 0$$

$$= \frac{u^{11}}{77} + 0$$



Strategy for u-sub.

- 1) Select u. (2) Find du and nolve fon dx in terms of du
- 3) Transform the original integral in terms of the variable x to an integral in terms of the variable u only.
  - (4) Apply the basic antiderivative table

$$\int u^{n} du = \frac{u^{n+1}}{n+1} + C ; n \neq -1$$

$$\int \frac{1}{u} du = \ln |u| + C.$$

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$$\int \sin(u) du = -\cos(u) + C$$

$$\int \cos(u) du = \sin(u) + C$$

$$\int e^{u} du = e^{u} + C$$

$$\int \frac{1}{1 + u^{2}} du = \arctan(u) + C$$

$$\frac{\text{E.g.}}{\text{G}}$$
 Find  $\left(\frac{7x^2+5}{4x^2+5}\right) \cdot x dx$ 

$$\frac{du}{dx} = 14x \rightarrow du = 14xdx \rightarrow dx = \frac{du}{14x}$$

(3) 
$$\int u^{2018} \cdot x \cdot \frac{du}{14x} = \int u^{2018} \frac{du}{14} = \frac{1}{14} \int u^{2018} \frac{du}{du} du$$
integral in terms of u only

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$$= \frac{1}{14} \cdot \frac{2019}{2019} + C = \frac{(7x^2 + 5)}{28266} + C$$

2019

$$E.g.$$
  $\left[x^2.\sin(x^3)dx\right]$ 

Let 
$$u = x^3$$
.  $du = 3x^2 dx \rightarrow dx = \frac{du}{3x^2}$ 

$$\int_{\mathbb{R}^{2}} \sin(u) \cdot \frac{du}{3x^{2}} = \frac{1}{3} \int_{\mathbb{R}^{2}} \sin(u) du$$

$$= -\frac{\cos(u)}{3} + C = -\frac{\cos(x^3)}{3} + C$$

Check: Take derivative of answer:

$$-\frac{\sin(x^3) \cdot 2x^2}{2} = x^2 \sin(x^3) - 3$$
same as integrand

u= lnx; du= 1 dx

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$$(5) \int y \sqrt{y^2 - 5} \, dy$$

$$\frac{7}{1+x^2} \int \frac{\arctan(x)}{1+x^2} dx du$$

$$u = x^{2} + 4$$
;  $du = 2xdx$ ;  $dx = \frac{du}{2x}$   

$$\int x \cdot e^{u} \cdot \frac{du}{2x} = \frac{4}{2} \int e^{u} du = \frac{1}{2} e^{x^{2} + 4} + C$$

$$\int u^2 du = \frac{u^3}{3} + C$$

$$=\frac{(\ln x)^3}{3}+C$$

$$\frac{1}{4} \int \frac{\sin(t)}{\cos^3(t)} dt$$

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$$\frac{1}{4} \left( \frac{\sin(t)}{\cos^3(t)} dt \right) = \int \frac{\sin(t)}{\cos(t)} \frac{1}{\cos^2(t)} dt$$

$$= \int u \, du = \frac{u^2}{2} + C = \frac{\tan^2(t)}{2} + C$$

2<sup>nd</sup> way: let u = con(t); du = -nin(t)dt

$$\int \frac{-du}{u^3} = -\int \frac{du}{u^3} = -\int u^{-3} du = -\frac{u^{-2}}{-2} + C$$

$$= \frac{1}{2u^2} + C = \frac{1}{2con^2(t)} + C$$

E.x. 
$$7$$
  $x \cdot (2x+5)$   $dx$ 

$$\left(8\right) \int \frac{dx}{\left(1+\sqrt{x}\right)^4}$$

(7) Let 
$$u = 2x + 5$$
. Then  $du = 2dx$ .

$$S_0$$
,  $dx = \frac{du}{2}$ 

$$\int_{\infty}^{\infty} \frac{2018}{u} \cdot \frac{du}{2} = \frac{1}{2} \int_{\infty}^{\infty} \frac{2018}{u} du$$
get or in terms of

$$u = 2x + 5$$
. So,  $x = \frac{u - 5}{7}$ 

$$\frac{1}{2}\left(\frac{u-5}{2}, u^{2018}\right) du = \frac{1}{4}\left((u-5)u^{2018}\right) du$$

$$\frac{1}{4} \left( \left( u^{2019} - 5u^{2018} \right) du = \frac{1}{4} \left( \frac{u^{2070}}{2020} - \frac{5u^{2019}}{2019} \right) + C$$

$$= \frac{u^{2070}}{8080} - \frac{5u^{2019}}{8076} + C$$