Answer: $\boxed{\dfrac{(2x+5)^{2020}}{8080} - \dfrac{5(2x+5)^{2019}}{8076} + C}$

⑧ $\displaystyle\int \frac{dx}{(\boxed{1+\sqrt{x}})^4}$ $\quad dx \rightarrow 2\sqrt{x}\,du$, $\quad 1+\sqrt{x} \rightarrow u$

Let $\boxed{u = 1+\sqrt{x}}$. Then $du = \dfrac{1}{2\sqrt{x}}dx$

So, $dx = 2\sqrt{x}\,du$

So, $\sqrt{x} = u-1$

$$\int \frac{2\sqrt{x}\,du}{u^4}$$

$$\int \frac{2(u-1)}{u^4}du = 2\int \frac{u-1}{u^4}du$$

$$= 2\int\left(\frac{1}{u^3} - \frac{1}{u^4}\right)du = 2\int\left(u^{-3} - u^{-4}\right)du$$

$$= 2\cdot\left(\frac{u^{-2}}{-2} - \frac{u^{-3}}{-3}\right) + C = -\frac{1}{u^2} + \frac{2}{3u^3} + C$$

$u \rightarrow 1+\sqrt{x}$

u- sub for definite integrals

E.g. $\displaystyle\int_0^4 \sqrt{2x+1}\,dx$

Let $u = 2x+1$. Then $du = 2\,dx$. So, $dx = \frac{du}{2}$

$$\int \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du$$

$1^{\underline{st}}$ way to proceed: continue finding the antiderivative, get the answer in terms of x, then use FTC-II, plug in bounds for x.

$u = 2x+1$ antiderivative

$$\frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{u^{\frac{3}{2}}}{3} = \boxed{\frac{(2x+1)^{\frac{3}{2}}}{3}}$$

$$\left.\frac{(2x+1)^{\frac{3}{2}}}{3}\right|_0^4 = \frac{(2\cdot4+1)^{\frac{3}{2}}}{3} - \frac{(2\cdot0+1)^{\frac{3}{2}}}{3}$$

$$= 9 - \frac{1}{3} = \boxed{\frac{26}{3}}.$$

$2^{\underline{nd}}$ way to proceed: Find the new bounds for u.

$$\frac{1}{2}\int u^{1/2}\,du$$

Bounds for x: $x = 0 \longrightarrow$ lower bound

$x = 4 \longrightarrow$ upper bound

Bounds for u: $u = 2x + 1$

$\longrightarrow$ lower bound for u: $2 \cdot 0 + 1 = 1$

upper bound for u: $2 \cdot 4 + 1 = 9$

$$\longrightarrow \frac{1}{2}\int_1^9 u^{1/2}\,du = \frac{1}{2} \cdot \frac{u^{3/2}}{3/2}\Bigg|_1^9$$

$$= \frac{u^{3/2}}{3}\Bigg|_1^9 = \frac{(9)^{3/2}}{3} - \frac{(1)^{3/2}}{3}$$

$$= 9 - \frac{1}{3} = \boxed{\frac{26}{3}}$$

E.x.

① $\displaystyle\int_0^1 (3t-1)^{50}\,dt$

② $\displaystyle\int_0^1 x\cdot e^{\boxed{-x^2}}\,dx$ $\rightarrow u$

③ $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$ $\rightarrow u$

④ $\displaystyle\int_0^{13} \frac{dx}{\sqrt[3]{(\boxed{1+2x})^2}}$ $\rightarrow u$

① Let $u = 3t-1$. Then $du = 3dt$. So, $dt = \frac{du}{3}$

$$\frac{1}{3}\int_{-1}^{2} u^{50}\,du$$

Plug $t=0$ in $u=3t-1$.
get lower bound for u is -1.
upper bound for u : 2

$$\frac{1}{3}\cdot\frac{u^{51}}{51}\Bigg|_{-1}^{2} = \frac{1}{153}\left(2^{51}+1\right)$$

② Let $u = -x^2$. Then $du = -2x\,dx$.

So, $dx = -\frac{du}{2x}$

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$$-\int \cancel{x}\cdot e^u \cdot \frac{du}{2\cancel{x}} = -\frac{1}{2}\int_0^{-1} e^u\,du = \frac{1}{2}\int_{-1}^{0} e^u\,du$$

$$= \frac{1}{2}e^u \Big|_{-1}^{0} = \frac{1}{2} - \frac{1}{2}e^{-1} = \boxed{\frac{1}{2} - \frac{1}{2e}}$$

3 $u = \frac{1}{x}$. Then $du = -\frac{1}{x^2}dx$.

So, $dx = -x^2\,du$

$$\int \frac{e^u}{\cancel{x^2}}(-\cancel{x^2}\,du) = -\int_1^{\frac{1}{2}} e^u\,du = \int_{\frac{1}{2}}^{1} e^u\,du$$

$$= e^u \Big|_{1/2}^{1} = \boxed{e - e^{1/2}}$$

4 Let $1+2x = u$. Then $du = 2\,dx$. So, $dx = \frac{du}{2}$

$$\int \frac{\frac{du}{2}}{\sqrt[3]{u^2}} = \frac{1}{2}\int \frac{du}{\sqrt[3]{u^2}} = \frac{1}{2}\int_1^{27} u^{-2/3}\,du$$

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$$\frac{1}{2} \cdot \frac{u^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} \Bigg|_{1}^{27} = \frac{3}{2} u^{\frac{1}{3}} \Bigg|_{1}^{27} = \frac{3}{2} \cdot (27)^{\frac{1}{3}} - \frac{3}{2}(1)^{\frac{1}{3}}$$

$$= \frac{9}{2} - \frac{3}{2} = \boxed{3}.$$