5.6. Integrals that involve Exp. and Log. Functions

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① Exp. Functions

<u>Recall</u>: $\frac{d}{dx}(e^x) = e^x$; $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

Let $a > 0$, $a \neq 1$ be any base

$\frac{d}{dx}(a^x) = a^x \cdot \ln a$; $\frac{d}{dx}(a^u) = a^u \ln a \cdot \frac{du}{dx}$

$$\boxed{\int e^x\,dx = e^x + C \quad ; \quad \int e^u\,du = e^u + C}$$

$a > 0$; $a \neq 1$ is any base

$$\boxed{\int a^x\,dx = \frac{a^x}{\ln a} + C \quad ; \quad \int a^u\,du = \frac{a^u}{\ln a} + C}$$

E.g. $\displaystyle\int 3^x\,dx = \frac{3^x}{\ln(3)} + C.$

$$\int e^{-x}\,dx = \int e^u(-du) = -\int e^u\,du$$
$$= -e^u + C = \boxed{-e^{-x} + C}$$

Let $u = -x$. Then $du = -dx$. So, $dx = -du$

E.g. $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

Let $u = 1 + e^x$. Then $du = e^x\,dx$.

So, $dx = \dfrac{du}{e^x}$

$$\int \cancel{e^x}\cdot\sqrt{u}\cdot\frac{du}{\cancel{e^x}} = \int u^{1/2}\,du = \frac{u^{3/2}}{3/2} + C$$

$$= \boxed{\frac{2}{3}\left(1+e^x\right)^{3/2} + C}$$

2 Integrals that involve Log Functions

Recall: $\frac{d}{dx}(\ln x) = \frac{1}{x}$; $\frac{d}{dx}(\ln u) = \frac{u'}{u}$

So, $$\boxed{\int \frac{1}{x}\,dx = \ln|x| + C \qquad \int \frac{1}{u}\,du = \ln|u| + C}$$

E.g. $$\int \frac{2x^3 + 3x}{x^4 + 3x^2}\,dx$$

Let $u = x^4 + 3x^2$. Then $du = (4x^3 + 6x)\,dx$
$= 2(2x^3+3x)\,dx$

So, $dx = \frac{du}{2(2x^3+3x)}$

$$\int \frac{\cancel{2x^3+3x}}{u} \cdot \frac{du}{2\cancel{(2x^3+3x)}} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C$$
$$= \boxed{\frac{1}{2}\ln|x^4+3x^2| + C}$$

$$\int \frac{dx}{x \ln x \ln(\ln x)} \qquad \to du$$

Let $u = \ln(\ln x)$. Then $du = \frac{1}{x \ln x} dx$

$$\int \frac{du}{u} = \ln|u| + C$$

$$= \boxed{\ln|\ln(\ln x)| + C}$$

E.g. $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)} dx$

Let $u = \cos(x)$. Then $du = -\sin(x)dx$

So $dx = \frac{du}{-\sin(x)}$

$$\int \frac{\cancel{\sin(x)}}{u} \cdot \left(\frac{du}{-\cancel{\sin x}}\right) = -\int \frac{du}{u} = -\ln|u| + C$$

$$= \boxed{-\ln|\cos x| + C} = \ln|(\cos x)^{-1}| + C$$

$$= \boxed{\ln|\sec x| + C}$$