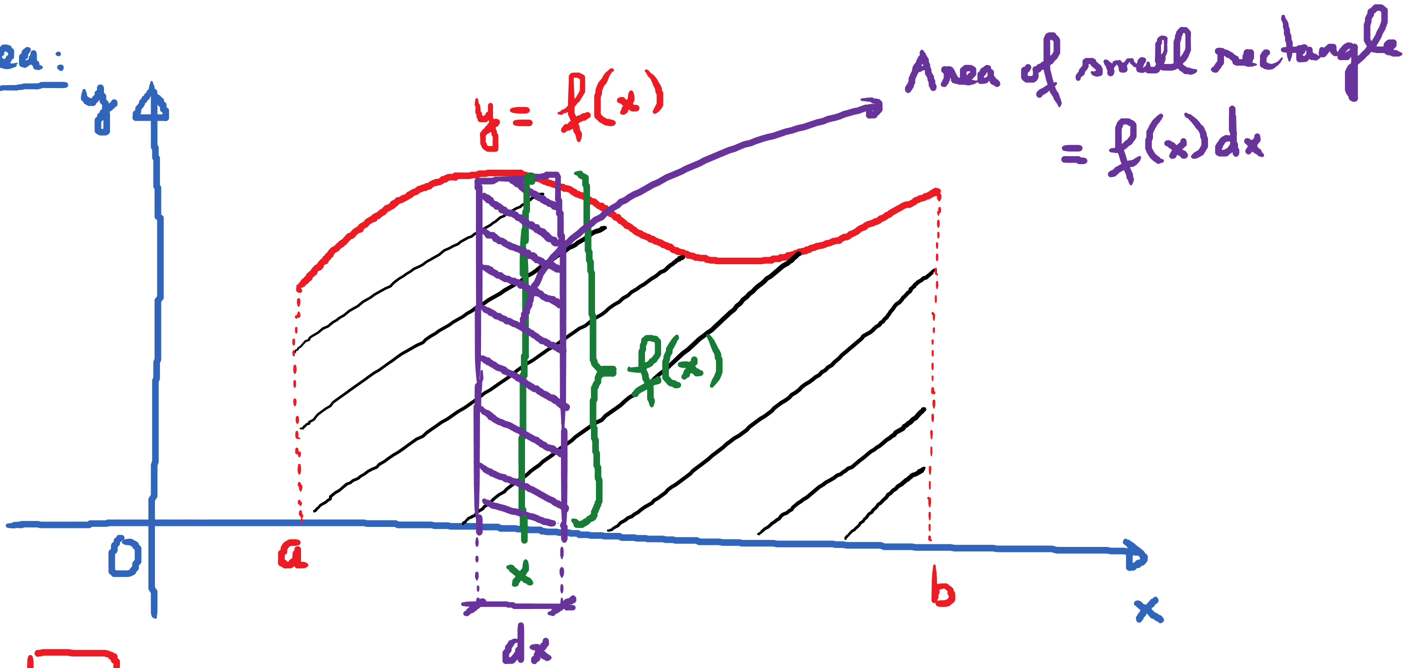
6.1. Areas Between Curves

Wednesday, August 15, 2018 7:35 AM

Idea:



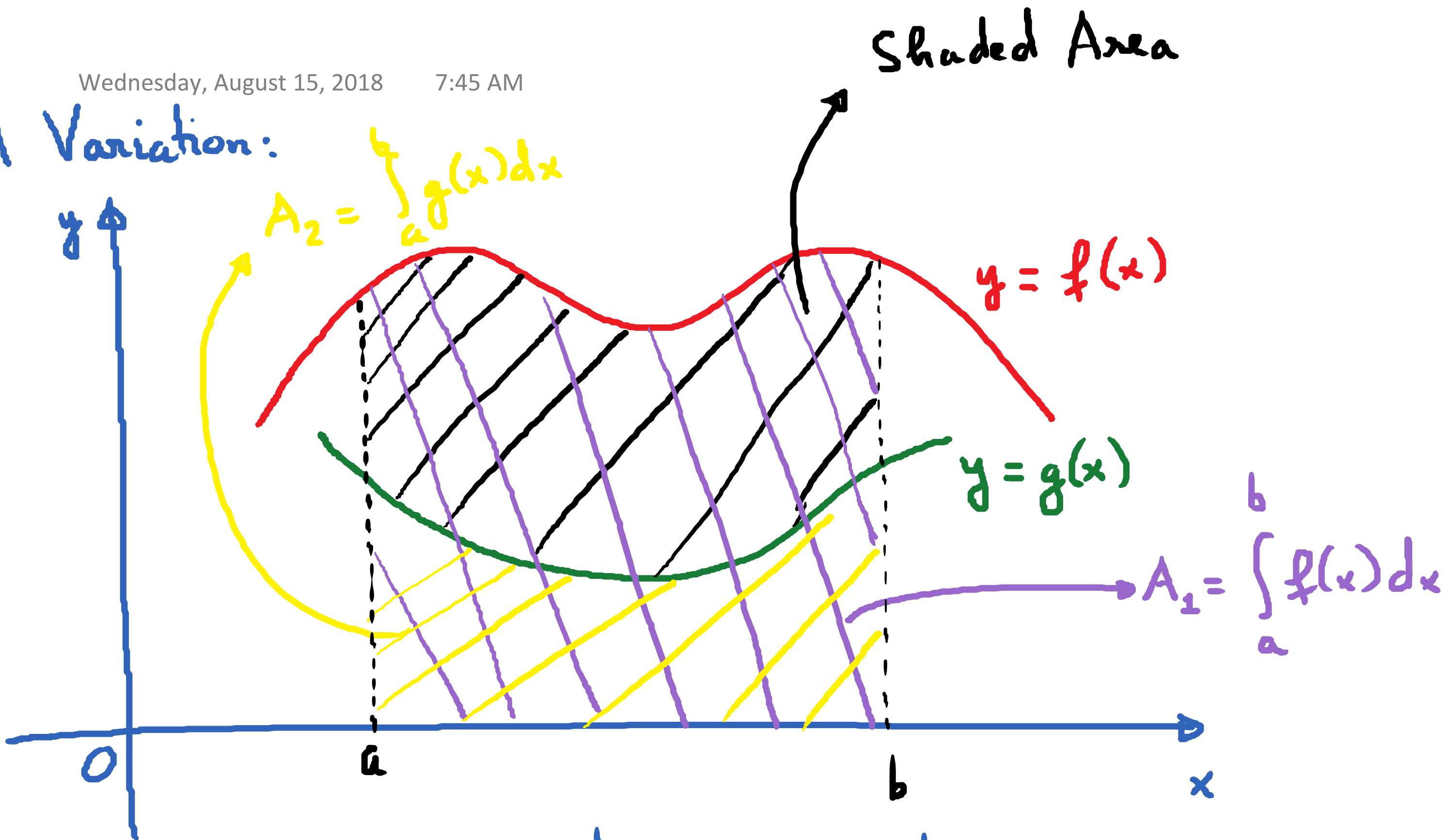
$$\boxed{\int_a^b} \boxed{f(x)} \boxed{dx} = \text{Shaded Area}$$

Infinite sum of small rectangular strips from a to b

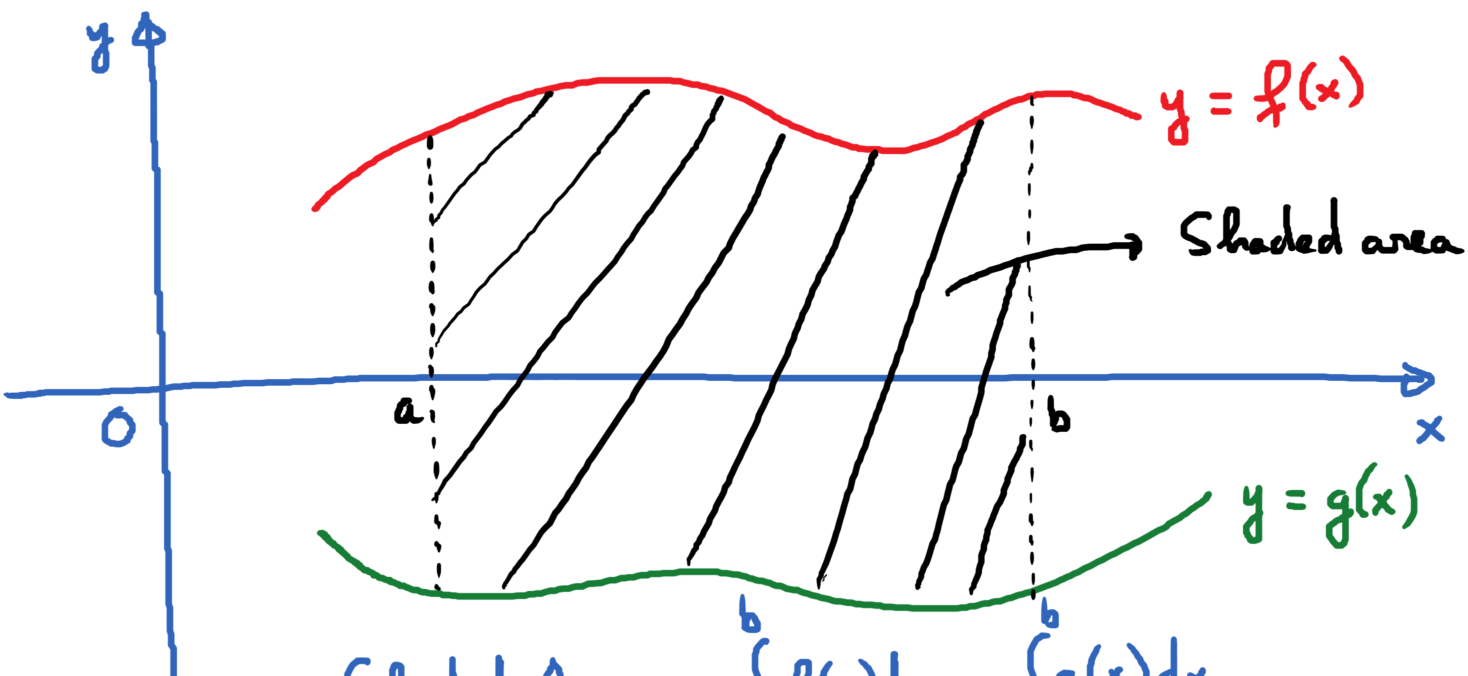
height of a small rectangular strip

width of a small rectangular strip

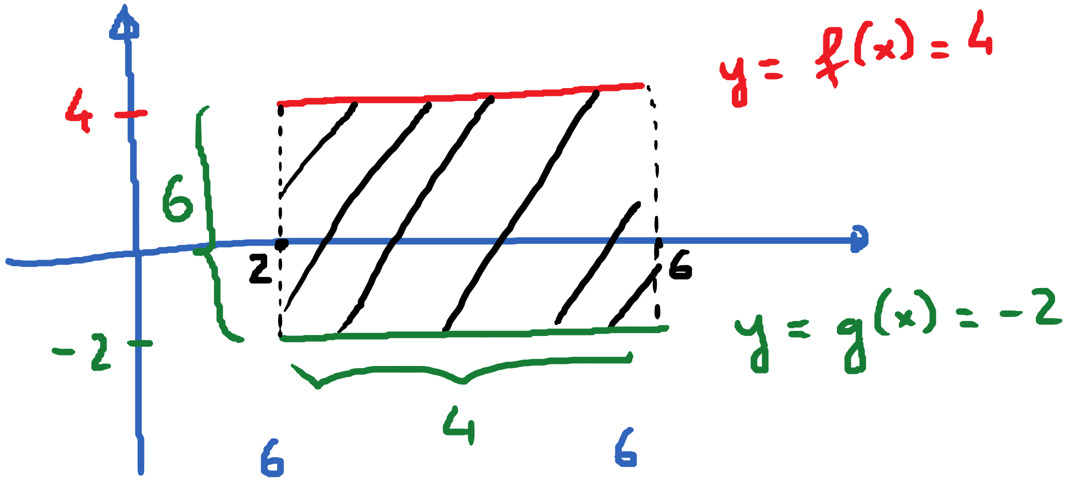
A Variation:



$$\text{Shaded Area} \stackrel{?}{=} \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$



$$\text{Shaded Area} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$



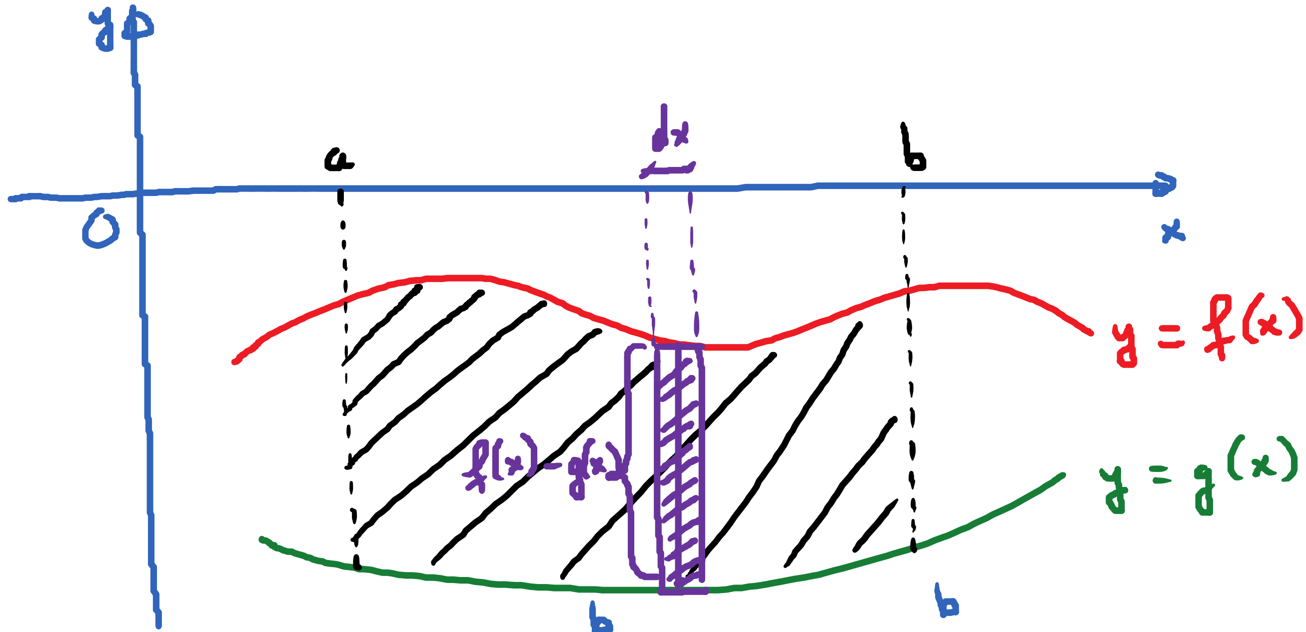
$\boxed{24}$

$$\int_2^6 f(x)\,dx + \int_2^6 g(x)\,dx$$

$$\int_2^6 4\,dx + \int_2^6 (-2)\,dx = 4\int_2^6 dx - 2\int_2^6 dx$$

$$= 4\cdot x\Big|_2^6 - 2\cdot x\Big|_2^6 = 4\cdot(6-2) - 2\cdot(6-2)$$

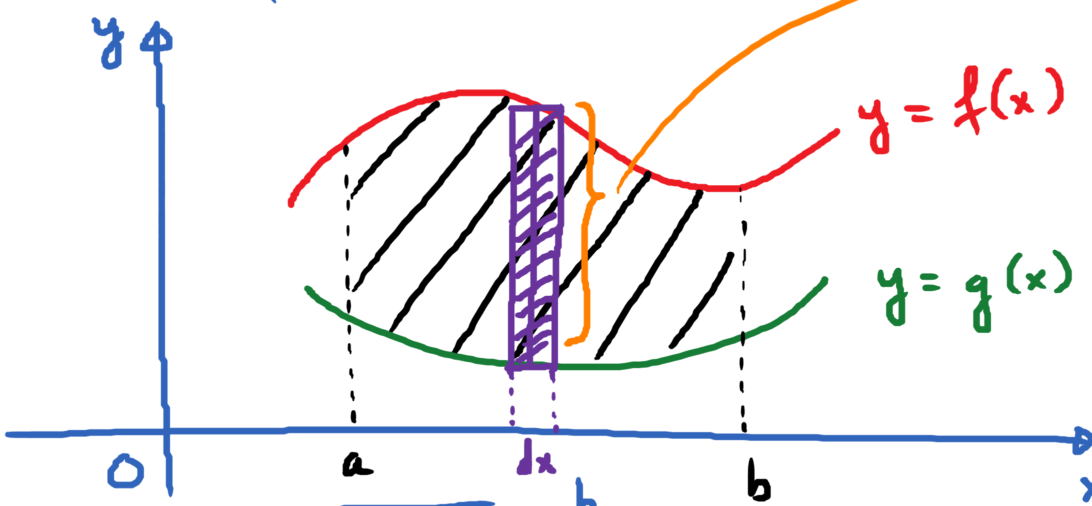
$$= 16 - 8 = \boxed{8}$$



$$\text{Shaded area} = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

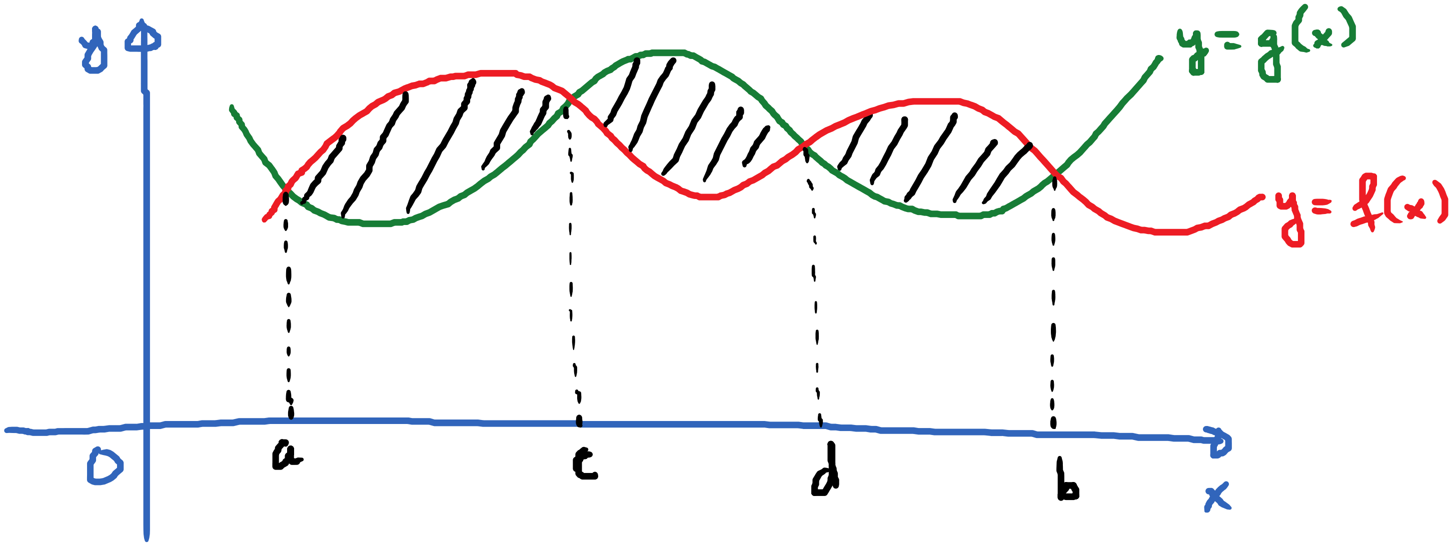
Formula for area between 2 curves.

height = top function - bottom function



$$\boxed{\text{Shaded Area} = \int_a^b \big(\underbrace{f(x)}_{\text{top curve}} - \underbrace{g(x)}_{\text{bottom curve}}\big)\,dx}$$

What if we have the following?



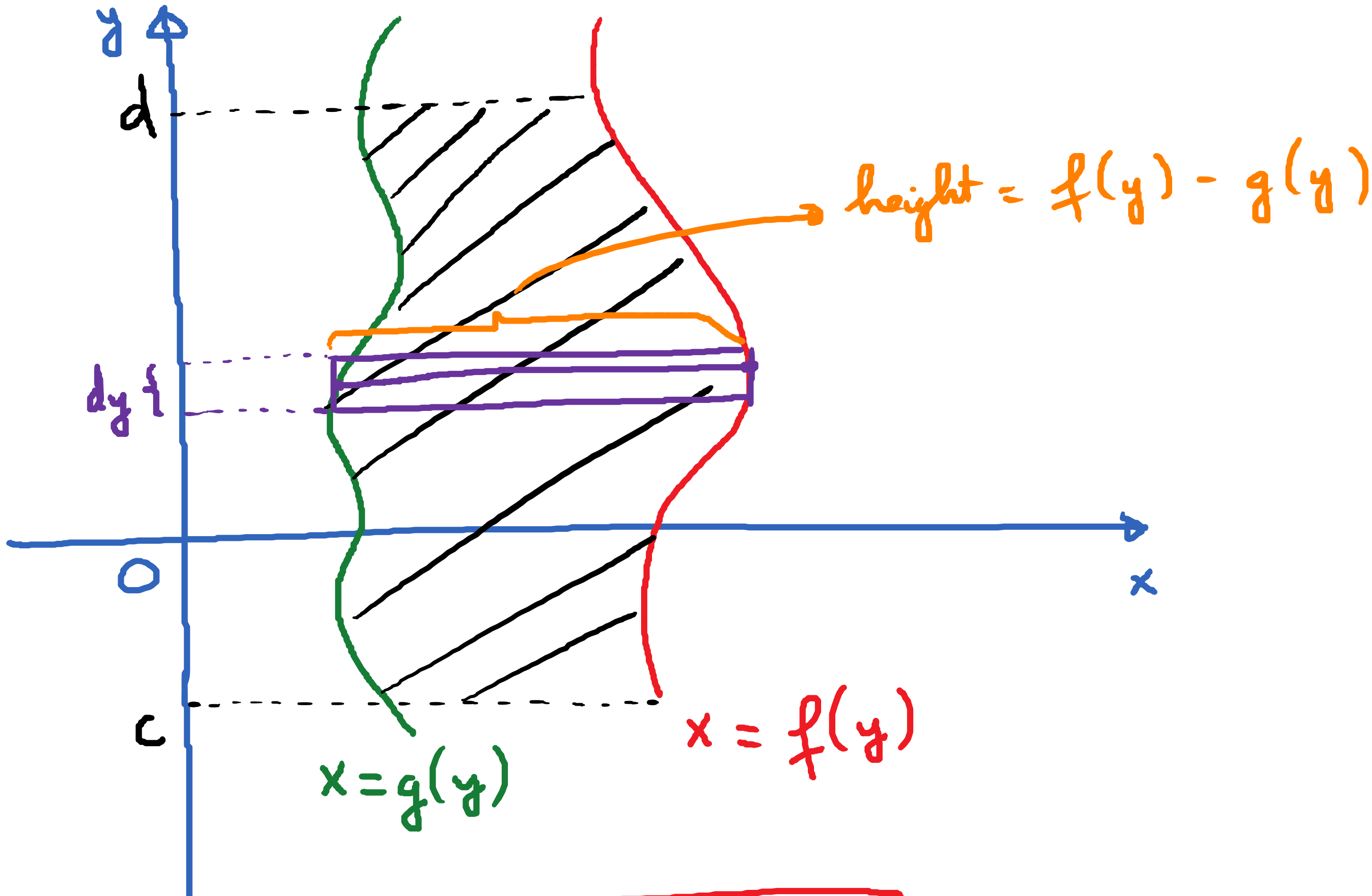
Step 1: Set $f(x) = g(x)$ to solve for the x-coordinates of the intersection points

Step 2:

$$\text{Area} = \int_a^c \big(f(x) - g(x)\big)\,dx + \int_c^d \big(g(x) - f(x)\big)\,dx$$

$$+ \int_d^b \big(f(x) - g(x)\big)\,dx$$

What if we have the following?



$$\text{Area} = \int_c^d \Big(\underbrace{f(y)}_{\text{rightmost curve}} - \underbrace{g(y)}_{\text{leftmost curve}}\Big)\,dy$$

E.g. #11. Review Sheet.

Step 1: Find points of intersection.

Set $f(x) = g(x)$.

So, $x^2 - 3 = 1 \longrightarrow x^2 = 4 \longrightarrow x = \pm 2$.

bounds for the integral

Step 2: Area $= \displaystyle\int_{-2}^{2} \Big(\underbrace{g(x)}_{\text{top curve}} - \underbrace{f(x)}_{\text{bottom curve}}\Big)\,dx$

$$= \int_{-2}^{2} \left[1 - (x^2 - 3)\right] dx = \int_{-2}^{2} (-x^2 + 4)\,dx$$

$$= \left(-\frac{x^3}{3} + 4x\right)\Bigg|_{-2}^{2} = \left(-\frac{8}{3} + 8\right) - \left(\frac{8}{3} - 8\right)$$

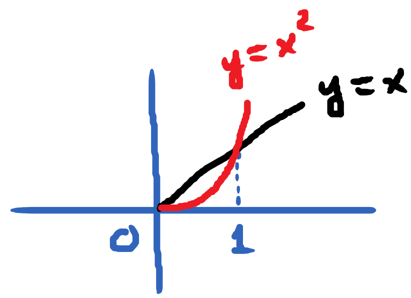
$$= 16 - \frac{16}{3} = \boxed{\frac{32}{3}}$$

#12

$$\int_{-1}^{1} (|x| - x^2)\,dx \longrightarrow |x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$= \int_{-1}^{0} (-x - x^2)\,dx + \int_{0}^{1} (x - x^2)\,dx$$

2^{nd} way: Shaded part on the right:



$$\text{Area} = 2 \cdot \int_0^1 (x - x^2)\,dx$$

$$= 2 \cdot \left(\frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_0^1$$

$$= 2 \cdot \left(\frac{1}{2} - \frac{1}{3} \right) = 2 \cdot \frac{1}{6} = \boxed{\frac{1}{3}}.$$