Name: __________________________

Student ID: __________________________

Section: __________________________

Instructor: __________________________

Math 2413 (Calculus I)
Practice Exam 1

Instructions:

- Each question is worth 5 points.
- Work on scratch paper will not be graded.
- No partial credit will be given for the multiple choice part and the short answer part.
- For questions 19 to 20, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Multiple Choice. Circle the correct answer for each question. Circle one choice only.

1. (Section 2.1) The height y in feet t seconds after a ball is thrown into the air with an initial velocity of 40 ft/s is given by
$$y = 40t - 16t^2.$$
Find the average velocity of the ball in the time interval $[2, 2.1]$.

a) 26.5 ft/s	b) 25.6 ft/s	c) 40 ft/s
d) -26.5 ft/s	e) -25.6 ft/s	f) -40 ft/s

2. (Section 2.2) For the function f whose graph is given in figure 1 below. Find the quantity
$$\lim_{x\to -3} f(x) + \lim_{x\to 0^+} f(x).$$

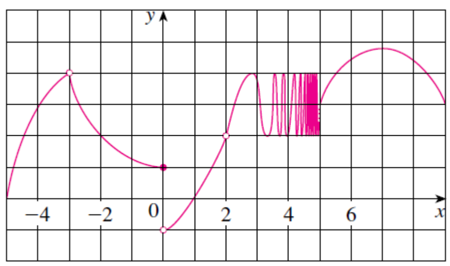

Figure 1: Figure for Question 2

a) -3	b) 3	c) 4
d) 5	e) 0	f) Does not exist

3. (Section 2.3) Find the limit $\displaystyle\lim_{\theta\to\pi} \frac{\sin\theta}{\tan\theta}$.

a) 0	b) -1	c) 1
d) $-\infty$	e) ∞	f) Does not exist

4. (Section 2.3) If $2x \le h(x) \le x^4 - x^2 + 2$ for all x, find $\displaystyle\lim_{x\to 1} h(x)$.

a) -1	b) -2	c) 1
d) 2	e) 0	f) Does not exist

5. (Section 2.4) Find the points at which the function is discontinuous and classify the type of discontinuity at each point for the function

$$g(t) = \frac{t+5}{t^2+9t+20}$$

a) Removable discontinuity at $t = -4$ and $t = -5$

b) Removable discontinuity at $t = -4$, Infinite discontinuity at $t = -5$

c) Removable discontinuity at $t = -5$, Infinite discontinuity at $t = -4$

d) Removable discontinuity at $t = -4$, Jump discontinuity at $t = -5$

e) Removable discontinuity at $t = -5$, Jump discontinuity at $t = -4$

f) Removable discontinuity at $t = 4$ and $t = 5$

6. (Section 2.4) Find the value of the constant c such that the function f is continuous on the interval $(-\infty, \infty)$.

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2. \end{cases}$$

a) $c = 2$

b) $c = 1$

c) $c = \frac{2}{3}$

d) $c = \frac{1}{3}$

e) $c = -2$

f) Does not exist

7. (Section 2.4) The graph of a function f is given in figure 2 below. State all the numbers at which f has a removable discontinuity. Choose the best answer.

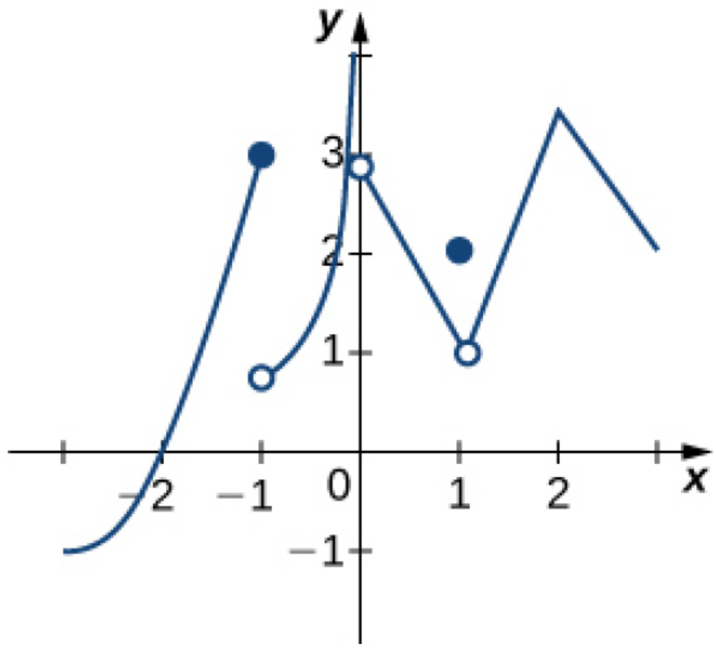

Figure 2: Figure for Question 7

a) -1 b) 0 c) 1

d) 2 e) $-1, 0$ f) $-1, 1$

8. (Section 3.1/3.2) The limit represents the derivative of a function f at a number a. Find f and a.

$$\lim_{h \to 0} \frac{\cos(\pi + h) + 1}{h}.$$

a) $f(x) = \cos(x + \pi)$, $a = 0$ b) $f(x) = \cos(x + \pi)$, $a = \pi$

c) $f(x) = \sin(x + \pi)$, $a = 0$ d) $f(x) = \cos(x)$, $a = 0$

e) $f(x) = \cos(x)$, $a = \pi$ f) $f(x) = \cos(x)$, $a = -\pi$

9. (Section 3.1/3.2) The tangent line to the curve $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$. Find $f'(4)$.

a) $\frac{1}{4}$ b) 2 c) 3

d) 4 e) -4 f) $-\frac{1}{4}$

10. (Section 3.1/3.2) The graph of a function g is given in figure 3 below. Arrange the following numbers in increasing order.

$$0 \qquad g'(-2) \qquad g'(0) \qquad g'(2) \qquad g'(4)$$

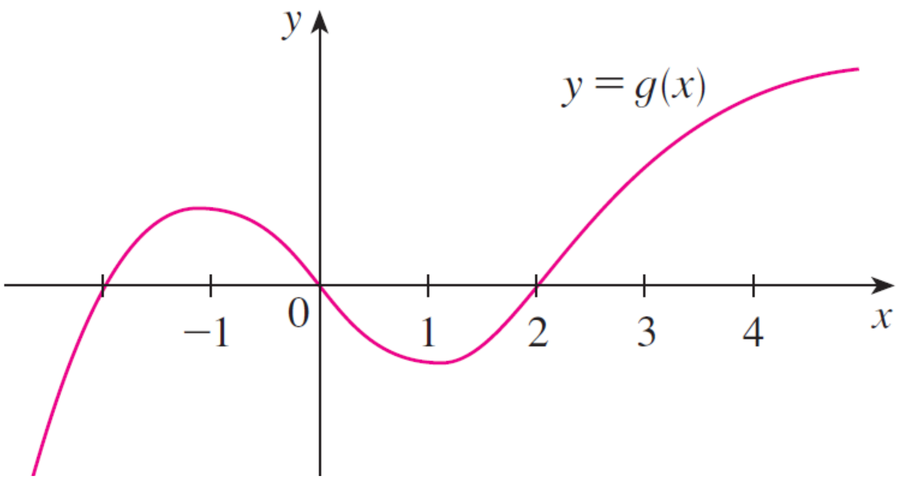

Figure 3: Figure for Question 12

a) $0 < g'(-2) < g'(0) < g'(2) < g'(4)$

b) $g'(4) < g'(2) < g'(0) < g'(-2) < 0$

c) $g'(-2) < 0 < g'(0) < g'(2) < g'(4)$

d) $g'(0) < 0 < g'(-2) < g'(2) < g'(4)$

e) $g'(0) < 0 < g'(4) < g'(2) < g'(-2)$

f) $g'(0) < 0 < g'(2) < g'(4) < g'(-2)$

11. (Section 3.3) Find an equation of the tangent line to the curve $y = \sqrt[4]{x}$ at the point $(1, 1)$.

a) $y = 4x - 3$

b) $y = -4x + 5$

c) $y = \frac{x}{4} + \frac{3}{4}$

d) $y = -\frac{x}{4} + \frac{5}{4}$

e) $y = \frac{3x}{4} + \frac{1}{4}$

f) $y = -\frac{3x}{4} + \frac{7}{4}$

12. (Section 3.1/3.2)The figure 4 below shows the graphs of f, f' and f''. Identify each curve.

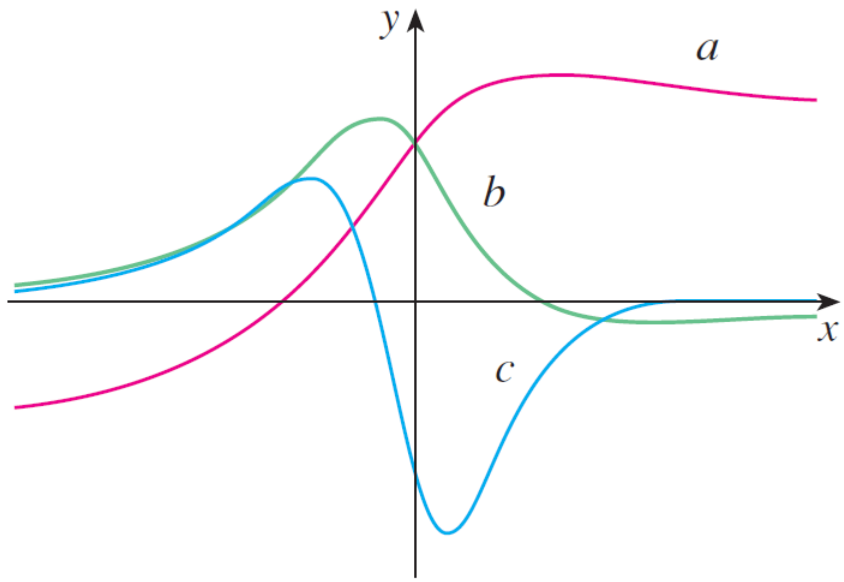

Figure 4: Figure for Question 14

a) f is a, f' is b, f'' is c b) f is a, f' is c, f'' is b c) f is b, f' is a, f'' is c

d) f is b, f' is c, f'' is a e) f is c, f' is a, f'' is b f) f is c, f' is b, f'' is a

13. (Section 3.3) The graphs of the functions F and G are given in figure 5 below. If $H(x) = \dfrac{F(x)}{G(x)}$, find $H'(7)$.

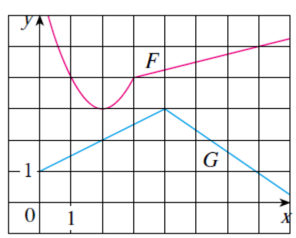

Figure 5: Figure for Question 15

a) $-\dfrac{39}{12}$ b) $\dfrac{39}{12}$ c) $-\dfrac{41}{12}$

d) $\dfrac{41}{12}$ e) $-\dfrac{43}{12}$ f) $\dfrac{43}{12}$

14. (Section 3.3) Find $\dfrac{dy}{dx}$ for the function $y = \dfrac{x^2+4x+3}{\sqrt{x}}$.

a) $\dfrac{1}{2}x^{-1/2} + 2x^{1/2} + \dfrac{3}{2}x^{1/2}$

b) $\dfrac{5}{2}x^{3/2} + 2x^{1/2} + \dfrac{3}{2}x^{1/2}$

c) $\dfrac{3}{2}x^{-1/2} + 2x^{-1/2} + \dfrac{3}{2}x^{-3/2}$

d) $\dfrac{3}{2}x^{1/2} + 2x^{-1/2} - \dfrac{3}{2}x^{-3/2}$

e) $\dfrac{3}{2}x^{5/2} + 2x^{3/2} - \dfrac{3}{2}x^{1/2}$

f) $\dfrac{3}{2}x^{5/2} + 2x^{3/2} + \dfrac{3}{2}x^{1/2}$

15. (Section 3.3/3.4) Find $f''(x)$ for the function $f(x) = x + \dfrac{1}{x}$.

a) $1 - \dfrac{1}{x^2}$

b) $1 - \dfrac{1}{x^3}$

c) $1 - \dfrac{2}{x^3}$

d) $\dfrac{2}{x^3}$

e) $-\dfrac{2}{x^3}$

f) $\dfrac{1}{2x^3}$

16. (Section 3.4) A particle moves according to the position function $s(t) = 3 - \dfrac{5}{t^2}$, $t > 0$. Find its acceleration function.

a) $\dfrac{10}{t^3}$

b) $-\dfrac{10}{t^3}$

c) $\dfrac{15}{t^4}$

d) $-\dfrac{15}{t^4}$

e) $\dfrac{30}{t^4}$

f) $-\dfrac{30}{t^4}$

Short Answer: Write your answers clearly for each question. No work will be graded. No partial credit.

17. (5 points) (Section 3.5) Find an equation of the tangent line to the graph of the function $f(x) = 4\sin(x) + 6\cos(x)$ at $x = 0$.

Answer:

18. (5 points) (Section 3.5) There are two x values for which the tangent line to the graph of $f(x) = x - 2\cos(x)$, $0 < x < 2\pi$ has slope 2. Find the sum of these two x-values.

Answer:

Essay: Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit. Scratch work will not be graded.

19. (5 points) (Section 2.3) Find the given limit **analytically**. (No credit will be given if you find the limit by making a table of values or graphing).

$$\lim_{x \to -4} \frac{\sqrt{x^2+9}-5}{x+4}$$

20. (5 points) (Section 3.2) Let $f(x) = 4x - 3x^2$.

(a) Find $f'(2)$ using the **definition of the derivative**. (**Any other method will receive NO credit**)

(b) Use the result from the previous part to find an equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 2$.

END OF EXAM