Name:________________________________

Student ID:____________________________

Section:_______________________________

Instructor:______________________________

Math 2413 (Calculus I)
Practice Exam 3

Instructions:

- Each question is worth 5 points.
- Work on scratch paper will not be graded.
- No partial credit will be given for the multiple choice part and the short answer part.
- For questions 19 to 20, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Multiple Choice. Circle the correct answer for each question. Circle one choice only.

1. (Section 4.3) Find the critical points of the function $f(x) = \dfrac{7x}{4x^2+1}$

   a) $x = \pm\dfrac{1}{4}$ b) $x = \pm\dfrac{1}{2}$ c) $x = -\dfrac{1}{4}$

   d) $x = -\dfrac{1}{2}$ e) $x = 0$ f) None of the above

2. (Section 4.3) Find the critical points of the function $f(x) = x^{3/5}(4-x)$.

   a) $x = 0$ b) $x = \dfrac{3}{2}$ c) $x = \dfrac{2}{3}$

   d) $x = -6$ e) $x = 0$ and $x = \dfrac{2}{3}$ f) $x = 0$ and $x = \dfrac{3}{2}$

3. (Section 4.4) Find the values of c guaranteed by the Mean Value Theorem for the function $f(x) = \ln(x)$ on the interval $[1, 4]$.

   a) $c = 0$ b) $c = \dfrac{4}{3}$ c) $c = \dfrac{3}{4}$

   d) $c = \dfrac{3}{\ln 4}$ e) $c = \dfrac{\ln 4}{3}$ f) Does not exist

4. (Section 4.5) Suppose the function f has second derivative $f''(x) = x^2(x-6)^4(x+2)^3(x^2-1)$. Moreover, f has horizontal tangent lines at $x = -3, \dfrac{1}{2}, 0, 2, -2$. Which value(s) correspond to a local minimum?

   a) -3 b) $\dfrac{1}{2}$ c) 2

   d) 0 and -2 e) -3 and 2 f) -3 and $\dfrac{1}{2}$

5. (Section 4.5) Suppose that a continuous function f has a derivative f' whose graph is shown in figure 1 below over the interval $(0, 9)$. Find the intervals over which f is decreasing. **Note: the graph is the graph of f'.**

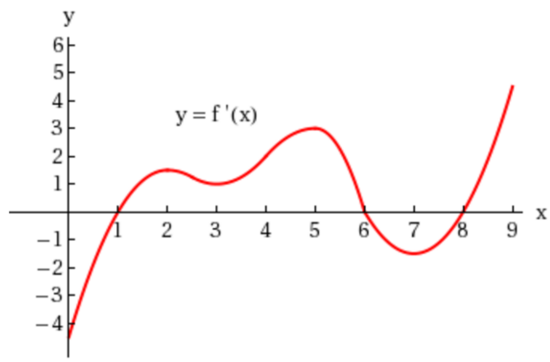

Figure 1: Figure for Question 5

a) $(0, 2) \cup (4, 6)$

b) $(2, 4) \cup (6, 8)$

c) $(0, 1) \cup (6, 8)$

d) $(1, 6) \cup (8, 9)$

e) $(2, 3) \cup (5, 7)$

f) $(0, 2) \cup (3, 5) \cup (7, 9)$

6. (Section 4.5) The graph of f' is given in figure 2. Find all the inflection points of the function f. **Note: the graph is the graph of f'.**

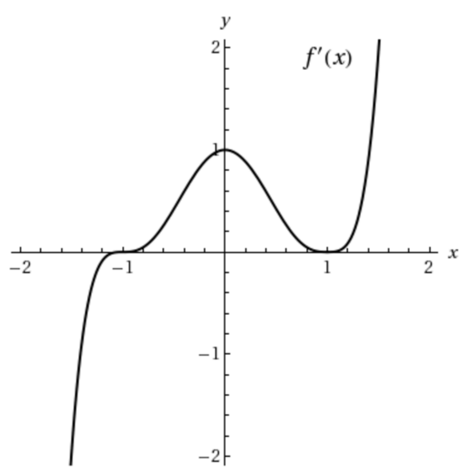

Figure 2: Figure for Question 6

a) $x = -1$

b) $x = 0$

c) $x = 1$

d) $x = -1$ and $x = 0$

e) $x = 0$ and $x = 1$

f) $x = -1$, $x = -0.4$ and $x = 0.4$

7. (Section 4.8) Find the limit $\lim_{x\to 0} \dfrac{\cos(mx) - \cos(nx)}{x^2}$.

a) 0 b) $\dfrac{m+n}{2}$ c) $\dfrac{m-n}{2}$

d) $\dfrac{m^2+n^2}{2}$ e) $\dfrac{m^2-n^2}{2}$ f) $\dfrac{n^2-m^2}{2}$

8. (Section 4.8) Evaluate the limit of the form ∞^0:

$$\lim_{x\to\infty} x^{1/\ln(x)}.$$

a) 0 b) 1 c) e

d) $\dfrac{1}{e}$ e) ∞ f) Does not exist

9. (Section 4.9) Suppose the tangent line to the curve $y = f(x)$ at the point $(2, 5)$ has equation $y = 9 - 2x$. Suppose that we use Newton's method to approximate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 2$. Find the second approximation x_2.

a) $-\dfrac{1}{2}$ b) $\dfrac{5}{2}$ c) $\dfrac{7}{2}$

d) $\dfrac{9}{2}$ e) $\dfrac{11}{2}$ f) None of the above

10. (Section 4.9) Suppose we apply Newton's method to the equation $x^2 - a = 0$, find the formula for the $(n+1)^{\text{th}}$ approximation x_{n+1} based on the n^{th} approximation x_n

a) $x_{n+1} = \dfrac{1}{2}x_n$ b) $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{a}{x_n}\right)$

c) $x_{n+1} = \dfrac{1}{2}\left(x_n - \dfrac{a}{x_n}\right)$ d) $x_{n+1} = \dfrac{1}{2}\left(3x_n + \dfrac{a}{x_n}\right)$

e) $x_{n+1} = \dfrac{1}{2}\left(3x_n - \dfrac{a}{x_n}\right)$ f) None of the above

11. (Section 4.10) Find the general antiderivative of the function $f(x) = \dfrac{(7+\sqrt{x})^2}{x}$.

a) $F(x) = x + \dfrac{21}{2}x^{2/3} + \dfrac{49}{x^2} + C$ b) $F(x) = x + 28\sqrt{x} - \dfrac{49}{x^2} + C$

c) $F(x) = x + 14\sqrt{x} + 49\ln|x| + C$ d) $F(x) = x + 28\sqrt{x} + 49\ln|x| + C$

e) $F(x) = x + \dfrac{7}{2}\sqrt{x} + 49\ln|x| + C$ f) $F(x) = \dfrac{x^2}{2} + \dfrac{7}{2}\sqrt{x} + 49\ln|x| + C$

12. (Section 4.10) Find the general antiderivative of the function $f(x) = \dfrac{49-x}{7-\sqrt{x}}$. (Hint: simplify first.)

a) $F(x) = 7x + \frac{2}{3}x^{3/2} + C$

b) $F(x) = 7x + \frac{3}{2}x^{2/3} + C$

c) $F(x) = 7x - \frac{2}{3}x^{3/2} + C$

d) $F(x) = 7x - \frac{3}{2}x^{2/3} + C$

e) $F(x) = 49x - \frac{2}{3}x^{3/2} + C$

f) $F(x) = 49x - \frac{3}{2}x^{2/3} + C$

13. (Section 5.1) Let R_n denote the right-endpoint Riemann sum using n subintervals. Compute the right sum R_6 for the function $f(x) = \dfrac{1}{x(x-1)}$ on the interval $[3, 6]$. (Round your answer to three decimal places.)

a) $R_6 = 0.191$

b) $R_6 = 0.192$

c) $R_6 = 0.193$

d) $R_6 = 0.194$

e) $R_6 = 0.195$

f) $R_6 = 0.1916$

14. (Section 5.1) Estimate the area under the curve in figure 3 by computing the left-endpoint Riemann sum, L_8.

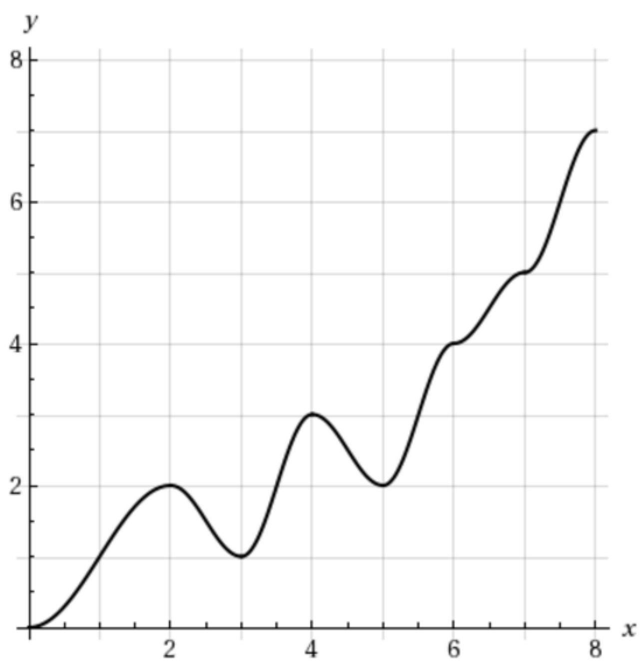

Figure 3: Figure for Question 14

a) 15

b) 16

c) 17

d) 18

e) 19

f) 20

15. (Section 5.2) Use the graph of the function g in figure 4 below to find $\int_0^7 g(x)dx$.

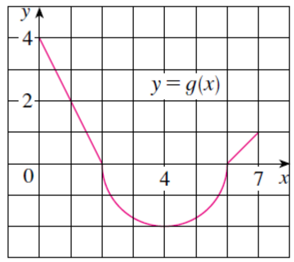

Figure 4: Figure for Question 15

a) $9 - 2\pi$ b) $9 - 4\pi$ c) $2\pi - 9$

d) $4.5 - 2\pi$ e) $4.5 - 4\pi$ f) $2\pi - 4.5$

16. (Section 5.2) Express the limit of the right Riemann sum R_n as a definite integral

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right) \log\left(\left(1 + \frac{i}{n}\right)^2\right)$$

a) $\int_0^1 x \log(x^2) dx$ b) $\int_0^1 x(\log(x))^2 dx$

c) $\int_1^2 x \log(x^2) dx$ d) $\int_1^2 x(\log(x))^2 dx$

e) $\int_1^2 \log(1 + x)^2 dx$ f) $\int_1^2 (1 + x) \log(1 + x)^2 dx$

Short Answer: Write your answers clearly for each question. No work will be graded. No partial credit.

17. (5 points) (Section 4.6) Evaluate the limit

$$\lim_{x\to-\infty} \frac{\sqrt{49x^2 - 1}}{x + 7}$$

Answer:

18. (5 points) (Section 4.7) The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. The area of printed material on the poster is fixed at 384 cm squared. Let x and y be the dimensions of the poster with the smallest area. Find $x + y$.

    Answer:

Essay: Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit. Scratch work will not be graded.

19. (5 points) (Section 4.5) Use the first derivative test to find the local maximum and local minimum of the function $f(x) = 2\ln(x) - 5\arctan(x)$. Show all work.

20. (5 points) (Section 4.5) Determine the intervals on which the function $g(x) = 6xe^{-x^2}$ is concave up/down. Show all work.

END OF EXAM