Name:______________________________

Student ID:______________________________

Section:______________________________

Instructor:______________________________

Math 2413 (Calculus I)
Practice Exam - Final

Instructions:

- Each question is worth 5 points.
- Work on scratch paper will not be graded.
- No partial credit will be given for the multiple choice part and the short answer part.
- For questions 19 to 20, show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Multiple Choice. Circle the correct answer for each question. Circle one choice only.

1. (Section 5.3) Find the derivative using the Fundamental Theorem of Calculus, Part 1.

$$\frac{d}{dx}\int_1^{e^x} \ln(u^4)du.$$

a) $4x$ b) $4e^x$ c) $4xe^x$

d) e^{4x} e) xe^{4x} f) x^4e^{4x}

2. (Section 5.3) Consider the graph of $F(x) = \int_0^x f(t)dt$ given in figure 1 below. (**Note: this is NOT the graph of f, it is the graph of F.**) Determine the interval(s) on which f is positive.

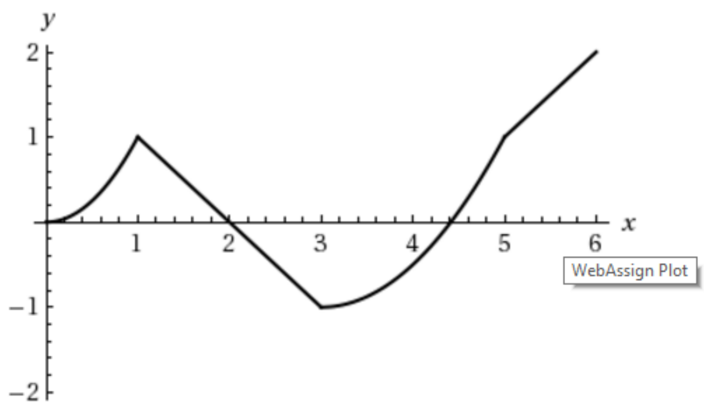

Figure 1: Figure for Question 2

a) $(0,2)\cup(4.4,6)$ b) $(0,1)\cup(3,6)$ c) $(1,3)$

d) $(0,3)\cup(5,6)$ e) $(0,6)$ f) $(1,3)\cup(5,6)$

3. (Section 5.5) Find the antiderivative $\int x(1-x)^{99}dx$.

a) $\frac{1}{99}(1-x)^{99} - \frac{1}{100}(1-x)^{100} + C$ b) $\frac{1}{100}(1-x)^{100} - \frac{1}{99}(1-x)^{99} + C$

c) $\frac{1}{100}(1-x)^{100} - \frac{1}{101}(1-x)^{101} + C$ d) $\frac{1}{101}(1-x)^{101} - \frac{1}{100}(1-x)^{100} + C$

e) $\frac{1}{101}(1-x)^{101} - \frac{1}{102}(1-x)^{102} + C$ f) $\frac{1}{102}(1-x)^{102} - \frac{1}{101}(1-x)^{101} + C$

4. (Section 5.6) Find the antiderivative $\int \frac{dx}{x(\ln(x))^2}$

a) $-\frac{1}{\ln(x)} + C$
b) $\frac{1}{\ln(x)} + C$
c) $-\frac{1}{(\ln(x))^2} + C$
d) $\frac{1}{(\ln(x))^2} + C$
e) $-\ln(\ln(x)) + C$
f) $\ln(\ln(x)) + C$

5. (Section 5.6) Find the antiderivative $\int e^{\tan x} \sec^2 x dx$

a) $e^{\sec x} + C$
b) $e^{\tan x \sec x} + C$
c) $\frac{1}{3} e^{\tan x} \sec^3 x + C$
d) $e^{\tan x} + C$
e) $e^{\cot x} + C$
f) $-e^{\cot x} + C$

6. (Section 5.6) Find the definite integral $\int_1^2 \frac{1 + 2x + x^2}{3x + 3x^2 + x^3} dx$

a) $\frac{1}{3} \ln(19)$
b) $3 \ln(19)$
c) $\frac{1}{3} \ln\left(\frac{18}{7}\right)$
d) $3 \ln\left(\frac{18}{7}\right)$
e) $\frac{1}{3} \ln\left(\frac{26}{7}\right)$
f) $3 \ln\left(\frac{26}{7}\right)$

7. (Section 5.7) Find the definite integral $\int_1^{\sqrt{3}} \frac{dx}{1 + x^2}$

a) $\frac{\pi}{3}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{12}$
e) $2\frac{\pi}{3}$
f) $4\frac{\pi}{3}$

8. (Section 5.7) Evaluate the indefinite integral $\int \frac{dx}{\sqrt{25 - x^2}}$.

a) $\sin^{-1}\left(\frac{x}{5}\right) + C$
b) $\frac{1}{5} \sin^{-1}\left(\frac{x}{5}\right) + C$
c) $5 \sin^{-1}\left(\frac{x}{5}\right) + C$
d) $\cos^{-1}\left(\frac{x}{5}\right) + C$
e) $\frac{1}{5} \cos^{-1}\left(\frac{x}{5}\right) + C$
f) $5 \cos^{-1}\left(\frac{x}{5}\right) + C$

9. (Section 5.7) Find the definite integral $\int_0^{5/2} \frac{\cos(\tan^{-1}(t))}{1 + t^2} dt$

a) $\frac{5}{\sqrt{27}}$
b) $-\frac{5}{\sqrt{27}}$
c) $\frac{5}{\sqrt{28}}$
d) $-\frac{5}{\sqrt{28}}$
e) $\frac{5}{\sqrt{29}}$
f) $-\frac{5}{\sqrt{29}}$

10. (Section 6.1) Find the value of c such that the area of the region bounded by $y = x^2 - c$ and $y = c^2 - x^2$ is 72.

a) 1 b) 2

c) 3 d) 4

e) 5 f) 6

11. (Section 6.1) Determine the area (in units squared) of the region between the two curves $f(x) = x^2 - 3$ and $g(x) = 1$. (See Figure 2)

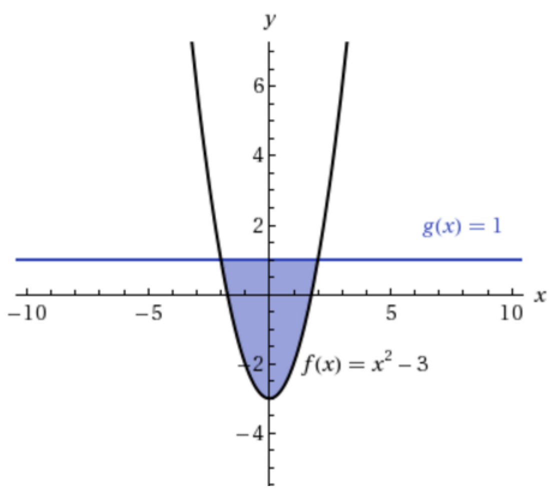

Figure 2: Figure for Question 11

a) 10 b) $\dfrac{31}{3}$ c) $\dfrac{32}{3}$

d) 11 e) $\dfrac{34}{3}$ f) $\dfrac{35}{3}$

12. (Section 6.1) Determine the area (in units squared) of the region between the two curves $y = |x|$ and $y = x^2$. (See Figure 3)

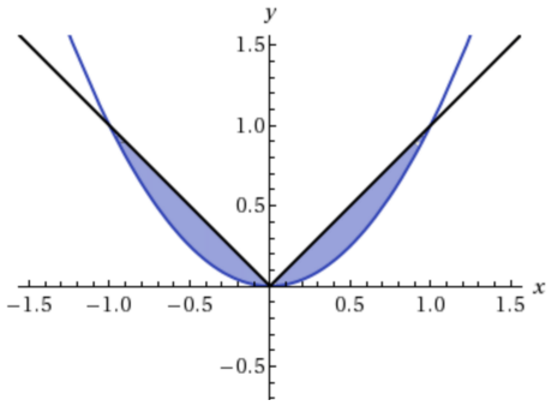

Figure 3: Figure for Question 12

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$

d) $\frac{1}{5}$ e) $\frac{1}{6}$ f) $\frac{1}{7}$

13. (Practice 1) Find the value of the constant c such that the function f is continuous on the interval $(-\infty, \infty)$.

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2. \end{cases}$$

a) $c = 2$ b) $c = 1$ c) $c = \frac{2}{3}$

d) $c = \frac{1}{3}$ e) $c = -2$ f) Does not exist

14. (Practice 2) Find the equation of the tangent line to the graph of the equation $x^2y^2 + 5xy = 36$ at the point $(4, 1)$.

a) $y = 4x - 15$ b) $y = -4x + 17$ c) $y = \frac{1}{4}x$

d) $y = -\frac{1}{4}x + 2$ e) $y = 5x - 19$ f) $y = \frac{1}{5}x + \frac{1}{5}$

15. (Practice 3) Find the limit $\lim_{x \to 0} \frac{\cos(mx) - \cos(nx)}{x^2}$.

a) 0 b) $\frac{m+n}{2}$ c) $\frac{m-n}{2}$

d) $\frac{m^2+n^2}{2}$ e) $\frac{m^2-n^2}{2}$ f) $\frac{n^2-m^2}{2}$

16. (Practice 3) Suppose the function f has second derivative $f''(x) = x^2(x-6)^4(x+2)^3(x^2-1)$. Moreover, f has horizontal tangent lines at $x = -3, \frac{1}{2}, 0, 2, -2$. Which value(s) correspond to a local minimum?

a) -3

b) $\frac{1}{2}$

c) 2

d) 0 and -2

e) -3 and 2

f) -3 and $\frac{1}{2}$

Short Answer: Write your answers clearly for each question. No work will be graded. No partial credit.

17. (5 points) (Practice 2) Find the derivative of the function $y = \sin(\sin(\sin(x)))$.

Answer:

18. (5 points) (Section 5.5) Find the definite integral $\int_0^a x\sqrt{a^2 - x^2}dx$.

Answer:

Free response: Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit. Scratch work will not be graded.

19. (5 points) (Section 5.6) Find the integrals using the substitution method(show all work, no credit if your answer is not justified by sufficient work):

(a) $\int e^{8x} dx$

(b) $\int \frac{\cos(x) - x\sin(x)}{x\cos(x)} dx$

20. (5 points) (Section 6.1) **Set up, but do not evaluate**, the definite integrals which give the area. Show the work in finding the upper and lower limit of the definite integrals.

   (a) The area of the shaded region bounded by $y = x^3 - 2x^2 + 2$ and $y = 3x^2 + 3x - 13$. (See Figure 4).

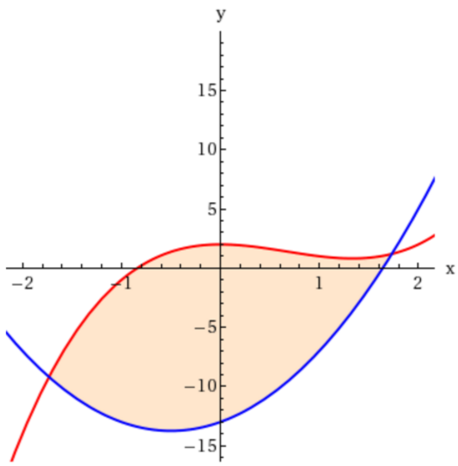

Figure 4: Figure for Question 23a

   (b) The area of the shaded region bounded by $y = 12 - x$, $y = \sqrt{x}$ and $y = 1$. (See Figure 5).

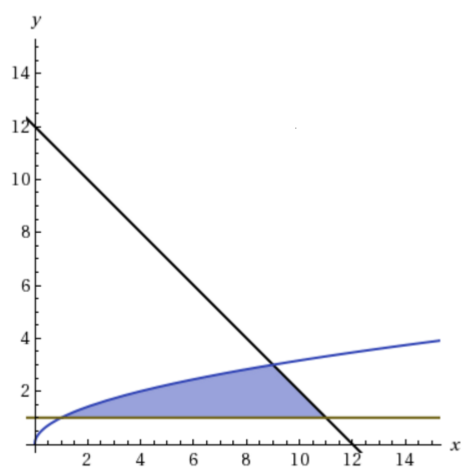

Figure 5: Figure for Question 23b

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