2.1. Preview of Calculus

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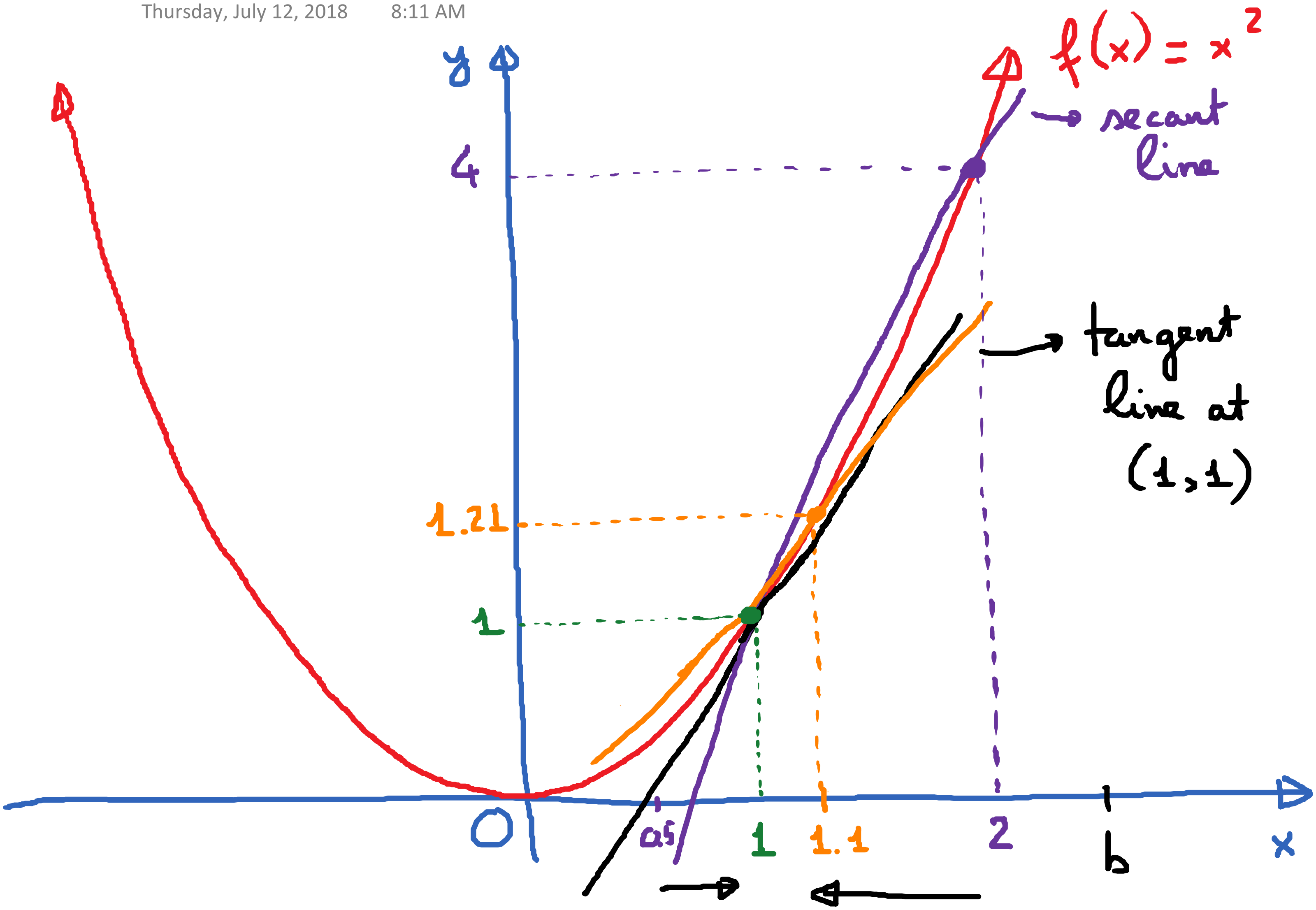
Goals: 1 The tangent line problem.

2 The area problem.

The tangent line problem

Problem: $f(x) = x^2$.

Question: Find the equation of the tangent line to the graph of this function at the point $(1,1)$.



Pick $x = 2 \rightarrow y = 4 \longrightarrow (2, 4)$

Slope of the secant line through $(1, 1)$ and $(2, 4)$: $\dfrac{4-1}{2-1} = \dfrac{3}{1} = 3$. $\left(\text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}\right)$

$m_{sec} = 3$.

x	$y = x^2$	m_{sec} = slope of the secant line through $(1,1)$ and (x,y)
2	4	3
1.1	1.21	$\dfrac{1.21 - 1}{1.1 - 1} = 2.1$
1.01	1.0201	$\dfrac{1.0201 - 1}{1.01 - 1} = 2.01$

If we let x gets close to 1 from the right $(x \to 1^+)$, it appears that the slopes of all the secant lines get closer and closer to the value of 2.

What if x gets close to 1 from the left?

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x	$y = x^2$	m_{sec}
0.5	0.25	$\dfrac{0.25 - 1}{0.5 - 1} = 1.5$
0.9	0.81	$\dfrac{0.81 - 1}{0.9 - 1} = 1.99$

It appears that the slopes of these secant lines also get closer and closer to 2.

<u>Reasonable conclusion:</u> Slope of the tangent line = 2

Equation of the tangent line?

Point (1, 1).

m ; (x_1, y_1)

Equation of the line with slope m and passes through the point (x_1, y_1):

$$\boxed{y - y_1 = m(x - x_1)}$$

In this situation : $m = 2$; point $(1, 1)$

$$\boxed{y - 1 = 2(x - 1)} \longrightarrow \text{Point-Slope form}$$

$$y - 1 = 2x - 2$$

$$\boxed{y = 2x - 1} \longrightarrow \text{Slope intercept form}$$

$(b, b^2) \rightarrow$ point on the graph $y = x^2$

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x	$y = x^2$	m_{sec} = slope of secant line that passes through $(1,1)$ and (b, b^2)
b	b^2	$\dfrac{b^2 - 1}{b - 1}$

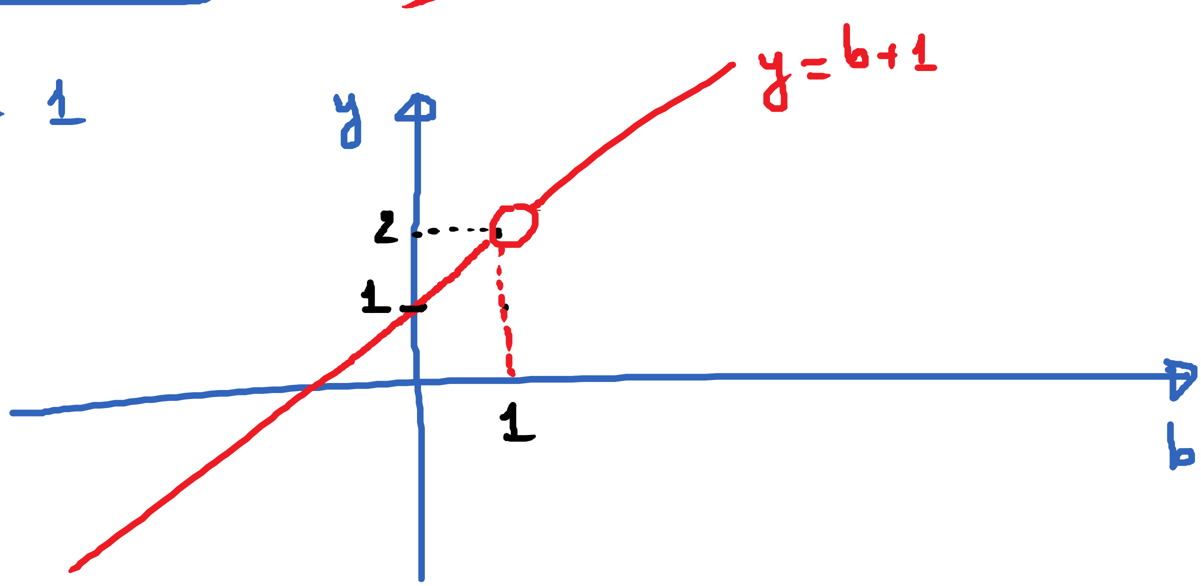
<u>Question:</u> What happens to $m_{sec} = \dfrac{b^2-1}{b-1}$ as the number b gets close 1 ?

$(b \to 1^+ \text{ or } b \to 1^-)$

$$\boxed{\frac{b^2-1}{b-1}} = \frac{\cancel{(b-1)}(b+1)}{\cancel{b-1}} = \boxed{b+1}$$

$b \neq 1$

$y = b + 1$



From the graph,

the quantity $\frac{b^2-1}{b-1}$ approaches 2 as

b gets close to 1.

Why do people care about the tangent line problem?

In physics, $f(t) = t^2$: position function of an object.

position at t = 2 seconds

position at t = 1 second.

from t = 1 s to t = 2 s

$$\frac{f(2) - f(1)}{2 - 1} = \frac{\text{distance obj. traveled}}{\text{time}}$$

= Average speed of obj on [1, 2]

Slope of secant line through (1,1); (2,4)

So, Slope of tangent line at $(1,1)$ = instantaneous speed at time $t = 1$ (s)

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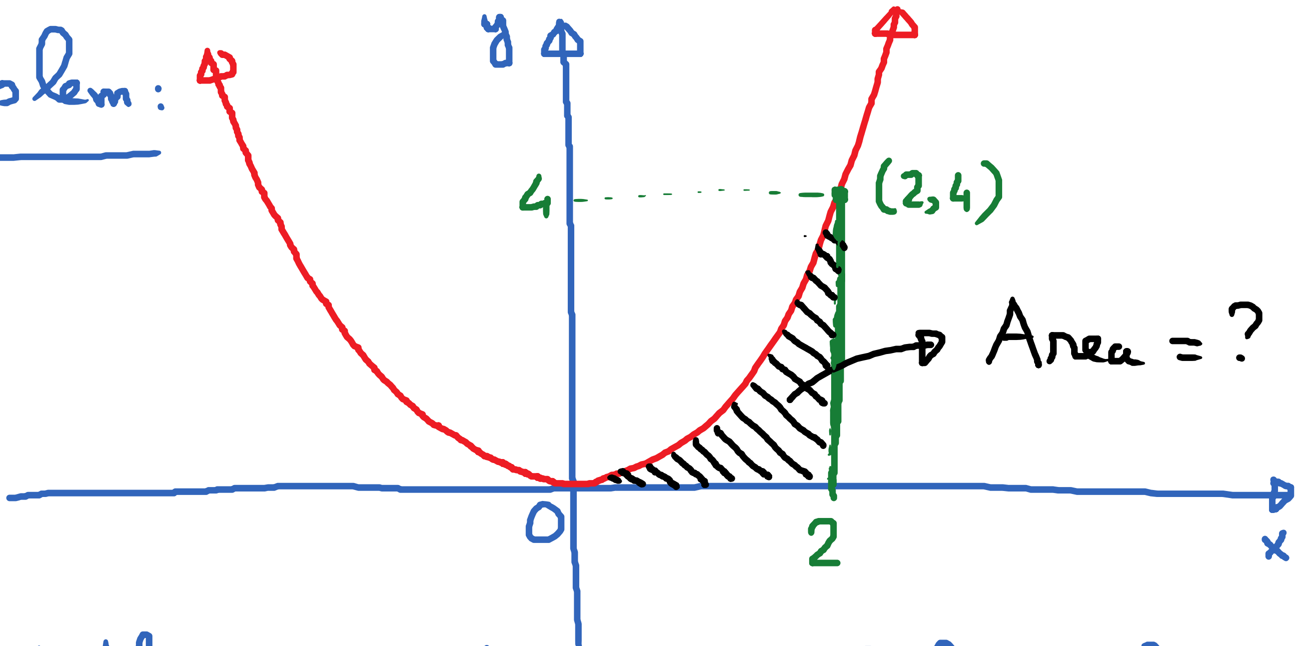
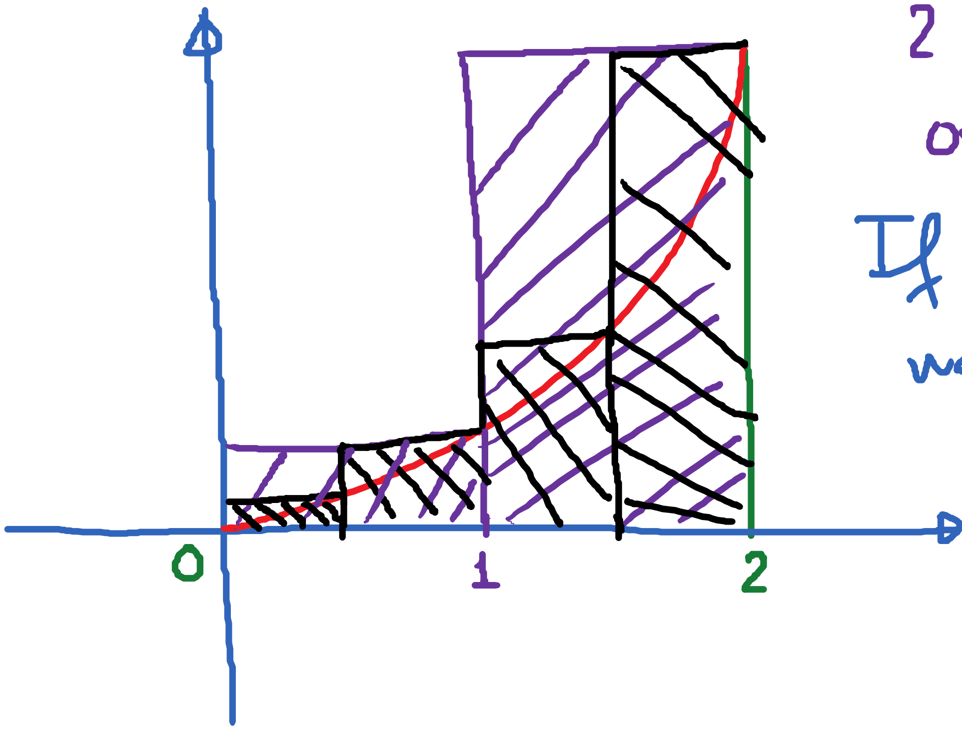
② Area Problem.

Problem:



Find the area under the graph of $y = x^2$ from $x = 0$ to $x = 2$.



Sum of areas of these
2 rectangles give an
over estimate of A.

If we use more rectangles
we get better and
better estimates.