2.2. The limit of a function.

Thursday, July 12, 2018 9:24 AM

Goals: (1) Discuss the concept of the limit of a function

(2) Find limits of functions by using graphs or by numerical method.

We have seen in 2.1. that m_{sec} approaches 2 when x approaches 1 from the right and from the left. This is a limiting process.

(depends on variable)

Variable ⟶ Quantity

Variable $\xrightarrow{\text{approach}}$ Fixed #

Quantity $\xrightarrow{\text{approach}}$ Fixed # ?

Definition of the limit of a function.

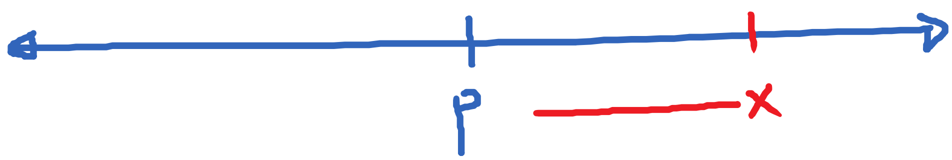
$y = f(x)$ is a function of x

We say that $f(x)$ approaches a number L as x approaches a number p to the right if $f(x)$ gets closer and closer to L as x gets closer and closer to p from the right.

$$\boxed{\lim_{x \to p^+} f(x) = L}$$

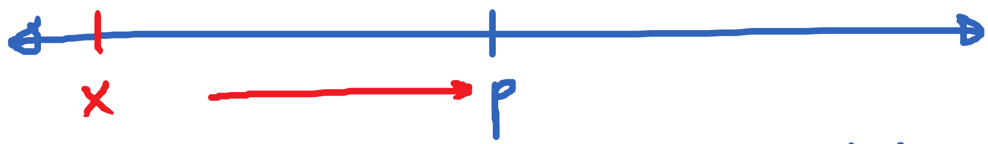
Right limit

Read as limit as x approaches p from the right of $f(x)$ is L.



In a similar way, we can define

$$\boxed{\lim_{x \to p^-} f(x) = L}$$ left limit



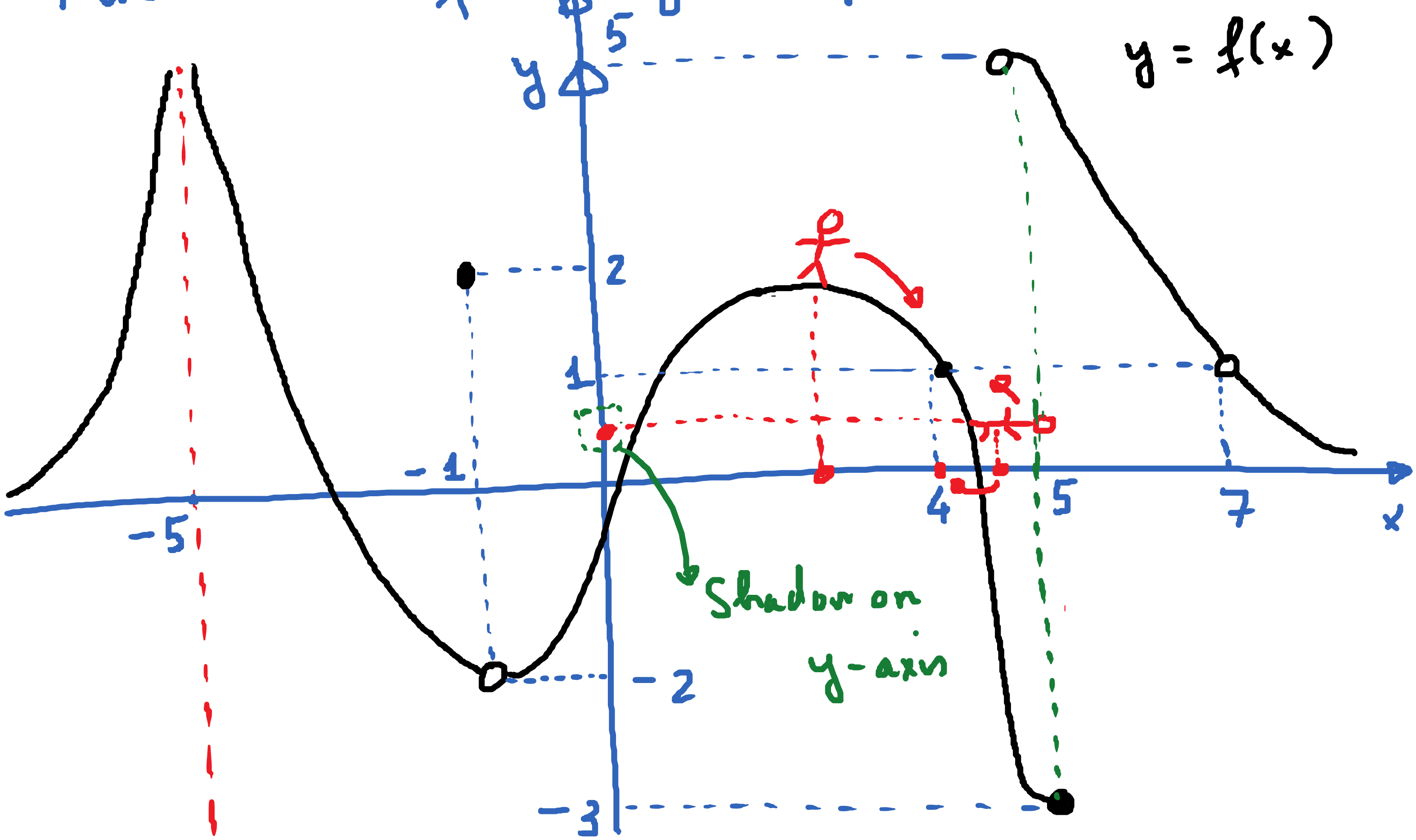
If $\lim_{x \to p^+} f(x) = \lim_{x \to p^-} f(x) = L$

(left limit = Right limit), we say that the limit of $f(x)$ as x approaches p exists, and

we write

$$\boxed{\lim_{x \to p} f(x) = L}$$

Find limits of f given graph of f.



$f(-1) = 2$; $f(7)$ DNE; $f(5)$ DNE

$x = 5$ is a V.A.

① $\lim_{x \to 4^+} f(x) = 1$ ② $\lim_{x \to 4^-} f(x) = 1$

③ $\lim_{x \to 4} f(x) = 1$

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Thursday, July 12, 2018 10:58 AM

(4) $\lim\limits_{x \to -1^+} f(x) = -2$

(5) $\lim\limits_{x \to -1^-} f(x) = -2$

(6) $\lim\limits_{x \to -1} f(x) = -2$

(7) $\lim\limits_{x \to 5^+} f(x) = 5$; $\lim\limits_{x \to 5^-} f(x) = -5$

$\lim\limits_{x \to 5} f(x)$ DNE.

(8) $\lim\limits_{x \to -5^+} f(x) = \infty$ (as x approaches -5 from right, $f(x)$ gets larger and larger.)

(9) $\lim\limits_{x \to -5^-} f(x) = \infty$

(10) $\lim\limits_{x \to -5} f(x) = \infty$

Thursday, July 12, 2018 11:05 AM

Find limits numerically.

$$\lim_{x \to 0^+} \frac{\sin(x)}{x} = 1$$

x	$\frac{\sin(x)}{x}$
0.1	0.99833
0.01	0.999983
0.001	0.999999...

$$\lim_{x \to 0^-} \frac{\sin(x)}{x} = 1$$

x	$\frac{\sin(x)}{x}$
-0.1	0.99833
-0.01	0.999983
-0.001	0.999999..

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$