2.3. Limit Laws

Monday, July 16, 2018 7:30 AM

Goals: (1) Find limits analytically using limit laws.

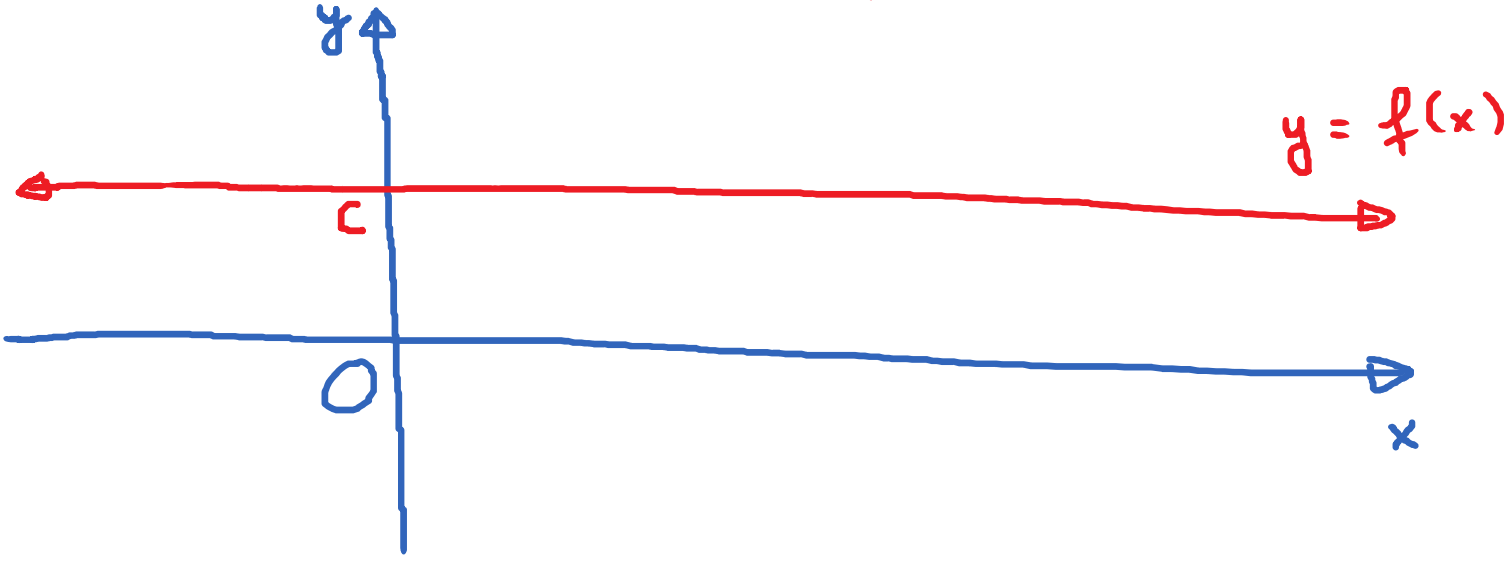
(2) Limits of the form $\frac{0}{0}$.

(3) Use the Squeeze Theorem to find limits

Basic Limits:

(1) c: constant. $f(x) = c$ for all x.

$f(x) = c$ constant function.

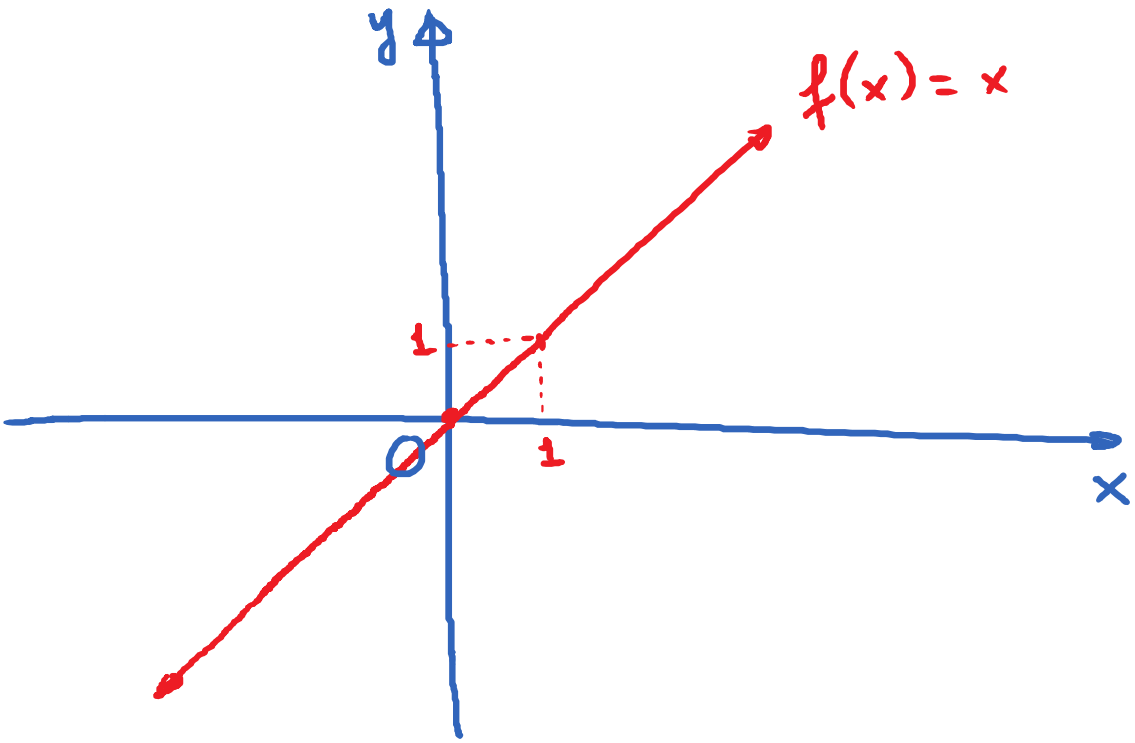


$$\lim_{x \to 2018} f(x) = c$$

In general, $\lim_{x \to a} f(x) = c$.

$$\boxed{\lim_{x \to a} [c] = c}$$

(2) Identity Function. $f(x) = x$



$$\lim_{x \to 2018} f(x) = 2018$$

In general, $\lim_{x \to a} f(x) = a$

$$\boxed{\lim_{x \to a} [x] = a}$$

Limit Laws: Given $\lim_{x \to a} f(x) = L$

$\lim_{x \to a} g(x) = M$

(1) **Sum Law:** $\lim_{x \to a} [f(x) + g(x)] = L + M$

E.g. $\lim_{x \to 5} (x + 7) = \lim_{x \to 5} [x] + \lim_{x \to 5} [7]$

Sum Law; 2nd basic limit; 1st basic limit

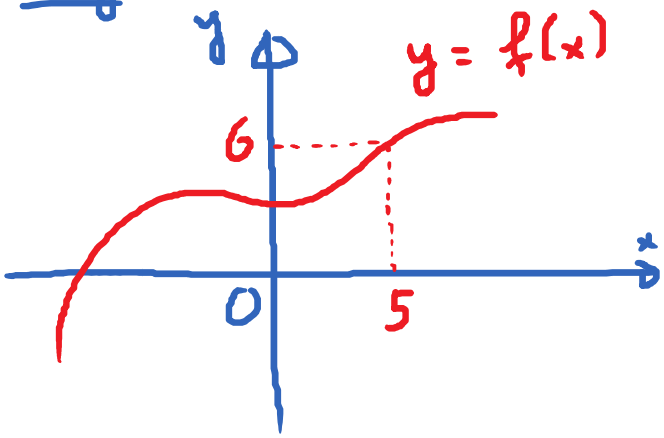
$= 5 + 7$

$= \boxed{12}$

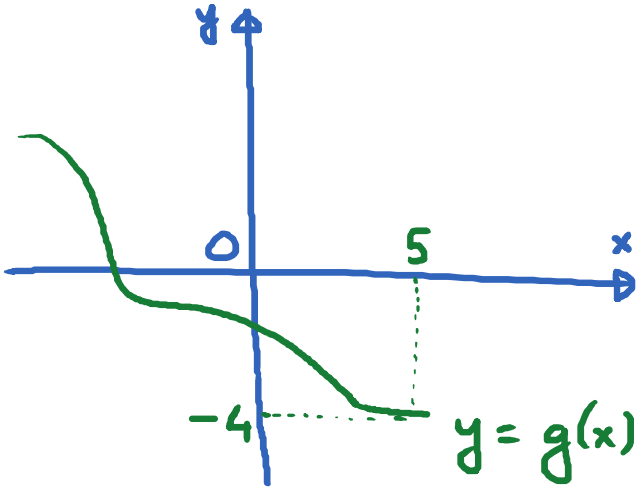
(2) Difference Law:

$$\lim_{x \to a} [f(x) - g(x)] = L - M.$$

E.g.



$$\lim_{x \to 5} [f(x)] = 6$$



$$\lim_{x \to 5} g(x) = -4$$

$$\lim_{x \to 5} [f(x) - g(x)] = 6 - (-4) = 10$$

↓

difference law

(c) Product Law:

$$\lim_{x \to a} [f(x)\,g(x)] = L \cdot M$$

E.g. $\lim_{x \to -2} [x^2] = \lim_{x \to -2} [x] \cdot \lim_{x \to -2} [x]$

$x^2 = x \cdot x$

product

2nd basic limit

$$= (-2) \cdot (-2) = 4$$

(Note: We can extend this to reason that

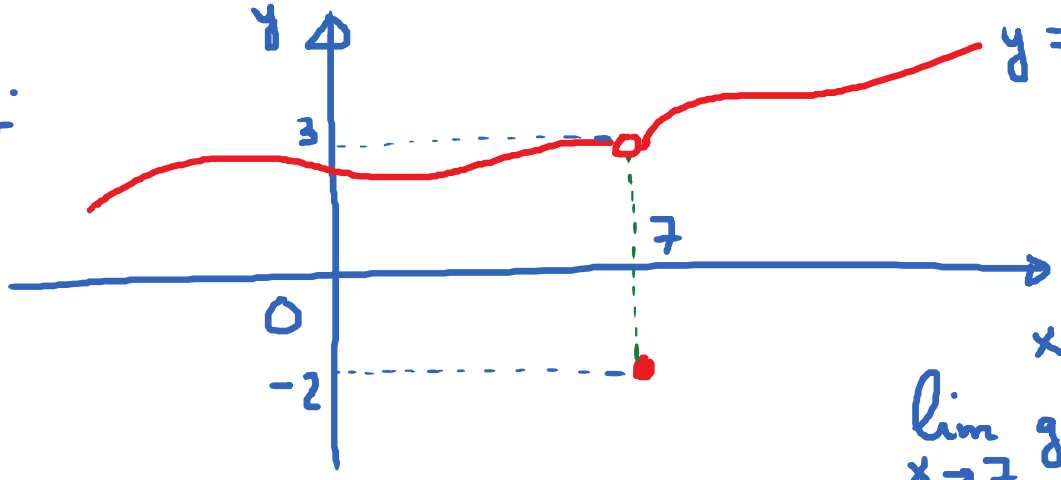
$$\lim_{x \to 6} [x^{2018}] = 6^{2018}$$

We can do this because of the product and the 2nd basic limit.)

(d) Constant Multiple Law:

$$\lim_{x \to a} [c \cdot f(x)] = c \cdot \lim_{x \to a} [f(x)]$$

E.g.



$\lim_{x \to 7} g(x) = 3$

Find $\lim_{x \to 7} [1000\, g(x)]$?

$= 1000 \cdot \lim_{x \to 7} [g(x)] = 1000 \cdot 3 = 3000$

Constant multiple law

(Find $1000 \cdot g(7) = 1000 \cdot (-2) = -2000$)

(e) Quotient Law: If $\lim_{x \to a} [g(x)] = M \neq 0$,

then $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{L}{M}$.

(f) Power Law:

$$\lim_{x \to a} [f(x)]^n = L^n$$

(g) Root Law:

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{L}$$

(provided that taking the root makes sense)

E.g. Find these limits and list all the laws that applied when you find the limits.

(1) $\lim_{x \to 1} (4x^2 - 2x + 3) = 5$ (Sum, Difference, product constant multiple)

(2) $\lim_{x \to -2} \sqrt{x^2 - 6x + 3} = \sqrt{19}$ (Sum, difference, product, constant multiple, root)

$x^2 - 6x + 3 \to 19$