Bottom Line: When we try to find $\lim_{x \to a} f(x)$, the first thing to do is to plug $x = a$ into the function. If we get a finite number, that is the answer! Done!

But for many situations, we do not get a finite #. It does not mean the limit DNE!

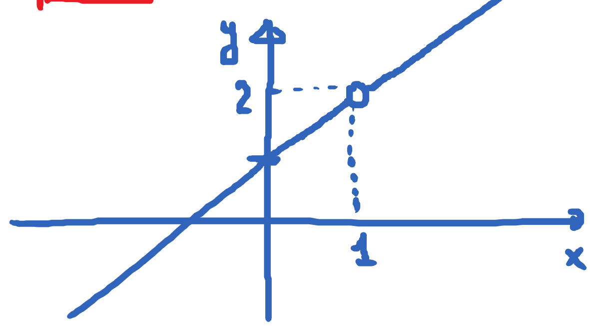
E.g. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0}$ → not a finite #

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \lim_{x \to 1} (x+1)$$

$$= \boxed{2}$$

(Reason why this works:

$f(x) = \frac{x^2 - 1}{x - 1}$



)

This suggests the following strategy to find limits of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ when we get $\frac{0}{0}$ if we plug in $x = a$.

Strategy: ① Factor top and bottom completely.

② Cancel the common factor(s)

③ Plug $x = a$ into the simplified expression.

Ex: ① $\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$

② $\lim_{h \to 0} \frac{(1+h)^2 - 1}{h}$

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① $$\lim_{x\to -3} \frac{x^2+4x+3}{x^2-9} \overset{\frac{0}{0}}{=} \lim_{x\to -3} \frac{(x+1)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}}$$

$$= \lim_{x\to -3} \frac{x+1}{x-3} = \frac{-2}{-6} = \boxed{\frac{1}{3}}$$

② $$\lim_{h\to 0} \frac{(1+h)^2-1}{h} \overset{\frac{0}{0}}{=} \lim_{h\to 0} \frac{h^2+2h+\cancel{1}\cancel{-1}}{h}$$

$$= \lim_{h\to 0} \frac{h^2+2h}{h} = \lim_{h\to 0} \frac{\cancel{h}(h+2)}{\cancel{h}}$$

$$= \lim_{h\to 0} (h+2) = \boxed{2}.$$

Some additional problems that involve $\frac{0}{0}$ limits

HW #4 $$\lim_{x\to 0} \frac{\sqrt{x+36}-6}{x} \overset{\frac{0}{0}}{=}$$

Multiply by the conjugate : $\sqrt{x+36}+6$

$$\lim_{x\to 0} \frac{\sqrt{x+36}-6}{x} \cdot \frac{\sqrt{x+36}+6}{\sqrt{x+36}+6}$$

$$= \lim_{x\to 0} \frac{(\underbrace{\sqrt{x+36}}_{A}-\underbrace{6}_{B})(\underbrace{\sqrt{x+36}}_{A}+\underbrace{6}_{B})}{x\left(\sqrt{x+36}+6\right)}$$

(Recall from algebra:

$$(A-B)(A+B) = A^2-B^2$$)

$$= \lim_{x\to 0} \frac{x+\cancel{36}-\cancel{36}}{x\left(\sqrt{x+36}+6\right)} = \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+36}+6\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+36}+6} = \frac{1}{\sqrt{36}+6}$$

$$= \frac{1}{6+6} = \boxed{\frac{1}{12}}$$

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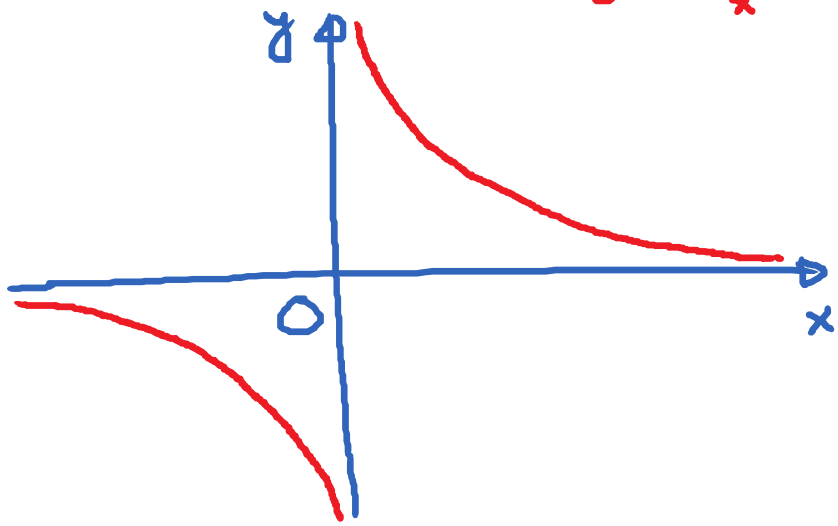
Limits of the form $\frac{K}{0}$ where K is a constant different from 0.

$y = \frac{1}{x}$

E.g.



$\lim_{x \to 0^+} \left(\frac{1}{x}\right) = \infty$; $\lim_{x \to 0^-} \left(\frac{1}{x}\right) = -\infty$.

$\downarrow$

$\frac{1}{0}$

In general, $\lim_{x \to a^+} \frac{K}{f(x)}$ and $\lim_{x \to a^+} f(x) = 0$

I $f(x) \to 0^+ \to$ answer: ∞

II $f(x) \to 0^- \to$ answer: $-\infty$

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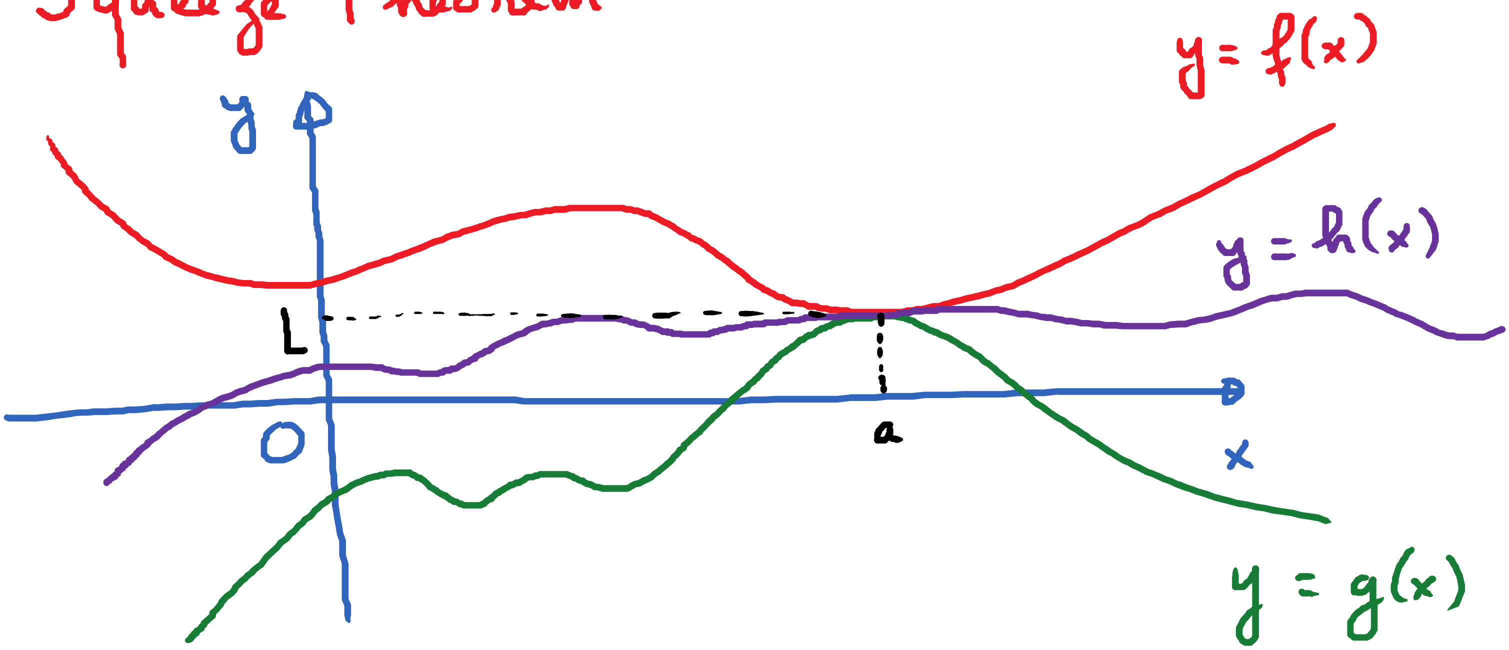
Squeeze Theorem.

Given:

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = L$$



$y = h(x)$ is "squeezed" in between f and g.

More precisely, $g(x) \le h(x) \le f(x)$ near a.

Q: $\lim_{x\to a} h(x) \overset{?}{=}$

Squeeze Thm says that $\lim_{x\to a} h(x) = L$