3.1. Definition of the Derivative

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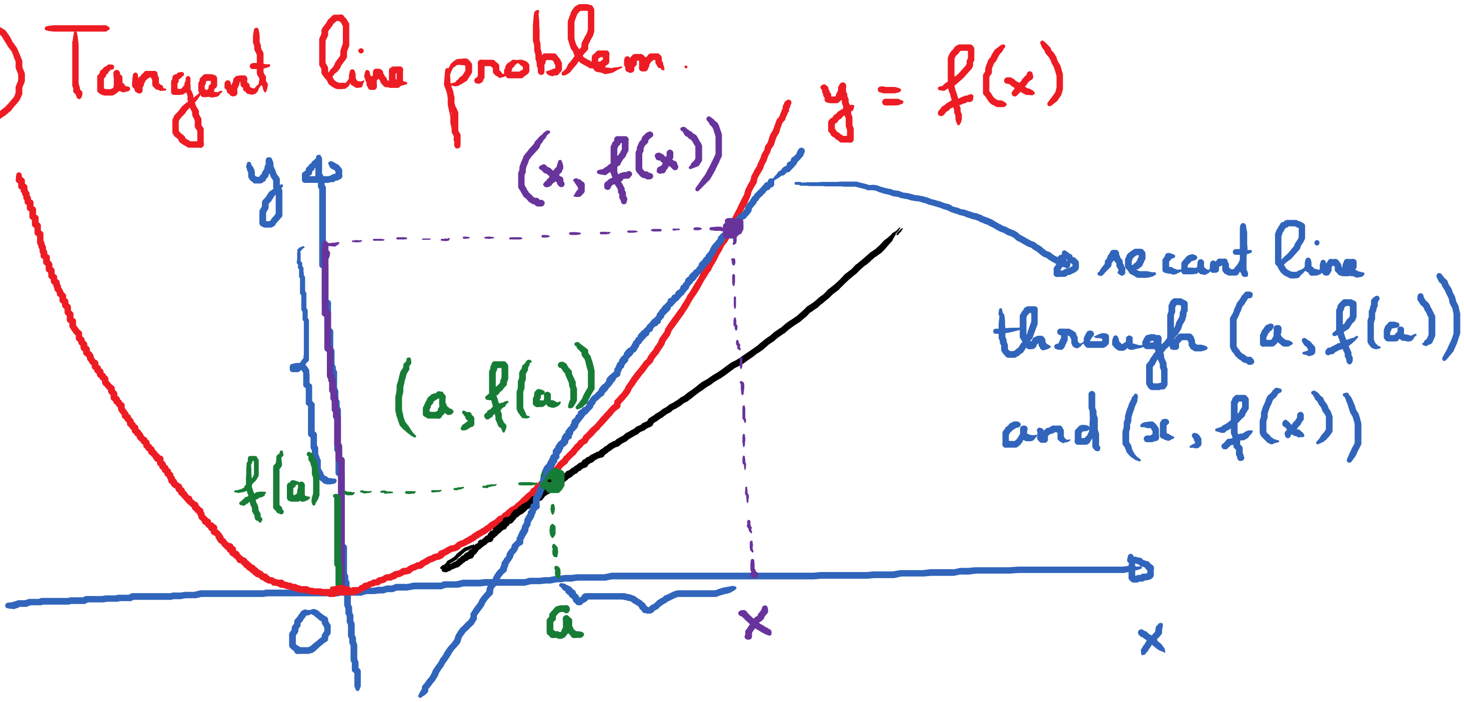
Goals: (1) Develop and apply the formula to calculate the slope of the tangent line to graph $y = f(x)$ at a given point

(2) Definition of the derivative of a function at a given point

(3) Calculate the derivative using this definition.

(1) Tangent line problem.



Slope of the secant line:

$$m_{sec} = \frac{f(x) - f(a)}{x - a}$$

So, the slope of the tangent line $m_{tangent}$ is

$$m_{tangent} = \lim_{x \to a} m_{sec}.$$

So, $$\boxed{m_{tangent} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}} \quad \text{I}$$

This formula gives you the slope of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$.

E.g. $f(x) = x^2$.

(a) Find the slope of the tangent line to the graph of f at the point $(3, 9)$.

a $\quad f(a)$

(b) Write the point-slope and the slope-intercept equation of the tangent line at $(3, 9)$

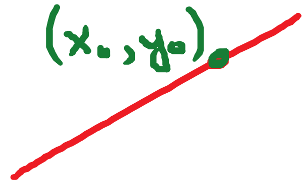
Sol: (a)

$$m_{\text{tangent}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 3} \frac{x^2 - 9}{x - 3} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$$

$$= \lim_{x \to 3} (x+3) = \boxed{6}$$



$m = \text{slope}$

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(b) Point - Slope Equation:

$$y - y_0 = m(x - x_0)$$

$m = 6$; $(x_0, y_0) = (3, 9)$

$$\boxed{y - 9 = 6(x - 3)}$$

Slope - Intercept Equation:

$$y - 9 = 6x - 18$$

$$\boxed{y = 6x - 9}$$

Ex. $f(x) = \dfrac{3}{x}$.

Find the slope and the equation of the tangent line to the graph of this function at the point where x-coordinate is 3. $\longrightarrow$ y-coord = 1.

$(3, 1)$

$$m_{tangent} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 3} \frac{\frac{3}{x} - \frac{1 \cdot x}{1 \cdot x}}{x - 3} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 3} \frac{\frac{3}{x} - \frac{x}{x}}{x - 3}$$

$$= \lim_{x \to 3} \frac{\boxed{\frac{3 - x}{x}}}{\boxed{\frac{x - 3}{1}}}$$

$$= \lim_{x \to 3} \frac{3 - x}{x} \cdot \frac{1}{x - 3}$$

$$\boxed{-\frac{1}{3}}$$

$$= \lim_{x \to 3} \frac{3 - x}{x(x - 3)}$$

$$= \lim_{x \to 3} \frac{-\cancel{[x - 3]}}{x\cancel{(x - 3)}} = \lim_{x \to 3} \frac{-1}{x}$$

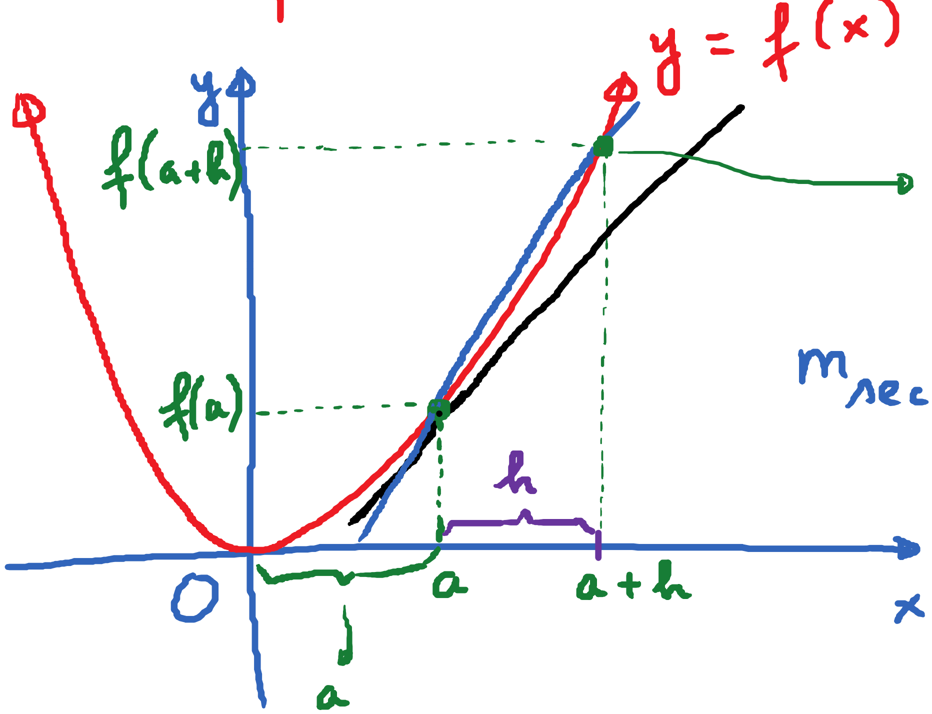
Pt - Slope equation:

$$y - 1 = -\frac{1}{3}(x-3)$$

Slope - intercept equation:

$$\boxed{y = -\frac{1}{3}x + 2}$$

An important variation of the formula for $m_{tangent}$:



$$m_{sec} = \frac{f(a+h) - f(a)}{h}$$

So, $m_{tangent} = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$