Second formula for $m_{tangent}$

$$\boxed{m_{tangent} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}} \quad \text{II}$$

Ex. $f(x) = \sqrt{x}$.

(a) Find the slope of the tangent line to the graph of f at the point whose x- coordinate is 1.
y-coord = 1

(b) Find the slope-intercept equation of the tangent line above.

(Use the second formula for part (a))

(a) $m_{sec} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$\sqrt{1+h}$

1

$$= \lim_{h \to 0} \frac{\boxed{f(1+h)} - \boxed{f(1)}}{h}$$

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$$\left(\frac{0}{0}\right)$$

$$\lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{1+h} - 1\right)\left(\sqrt{1+h}+1\right)}{h\left(\sqrt{1+h}+1\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{1}+h-\cancel{1}}{h\left(\sqrt{1+h}+1\right)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{1+h}+1\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{1+h}+1} = \boxed{\frac{1}{2}}$$

(b) Pt-Slope equation: $y - 1 = \frac{1}{2}(x-1)$

Slope-Intercept: $\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

So far, slope of tangent line to graph of $y = f(x)$ at point $(a, f(a))$:

$$m_{sec} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

VID: The derivative of the function $y = f(x)$ at the point where $x = a$; denoted by,

$$\underbrace{f'(a)}_{\text{read as } f \text{ "prime" at } a} \quad \text{or} \quad \underbrace{\left.\frac{dy}{dx}\right|_{x=a}}_{\text{read as } dy \text{ over } dx \text{ at } x=a}$$

(Leibnitz)

is defined as:

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

(provided the limit exists)

Note: (1) $f'(a)$ = slope of the tangent line to f at pt $(a, f(a))$

→ this is the geometric meaning of the derivative

(2) $f'(a)$ = instantaneous rate of change of f at $x = a$.

E.g. $f(x) = 2x + 5$

(1) Calculate $f'(3)$ using the limit definition.

(2) Calculate $f'(3)$ by interpreting it as slope of tangent line

(1) $$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x\to 3} \frac{2x + 5 - 11}{x - 3} = \lim_{x\to 3} \frac{2x - 6}{x - 3}$$

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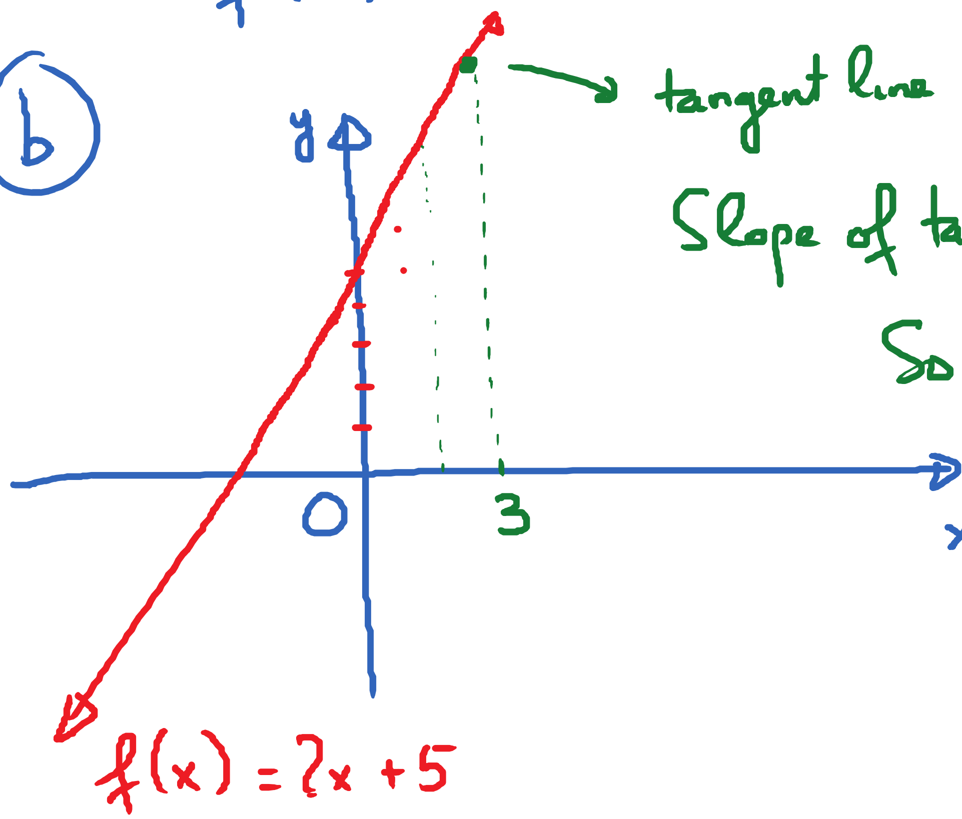
$$= \lim_{x \to 3} \frac{2\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x \to 3} (2) = \boxed{2}$$

$$f'(3) = 2.$$

b



tangent line at $x = 3$ is $y = 2x + 5$

Slope of tangent line $= 2$

So, $f'(3) = 2$.

Ex. $f(x) = 8 - x - x^2$.

Find $f'(0)$ using the definition.

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{\quad}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} - h - h^2 \cancel{- 8}}{h}$$

$$= \lim_{h \to 0} \frac{-h - h^2}{h} = \lim_{h \to 0} \frac{-\cancel{h}(1 + h)}{\cancel{h}}$$

$$= \lim_{h \to 0} -(1 + h) = \boxed{-1}$$

So, $\boxed{f'(0) = -1}$