

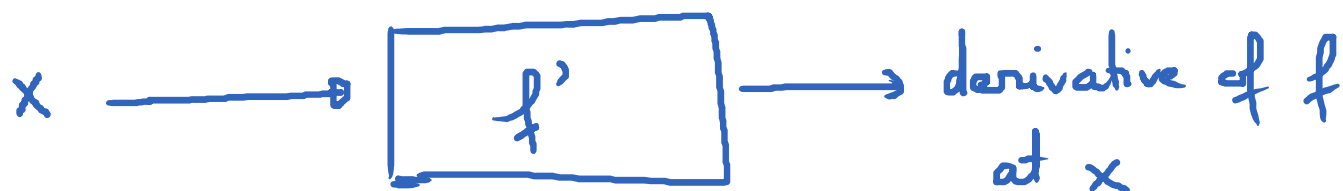
3.2. The derivative as a function

Tuesday, July 17, 2018

10:10 AM

The formula $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ gives us the derivative of f at a particular point $x = a$.

We want : to have a formula that can give us the derivative at arbitrary points.



First step: replace a by x in the definition:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

This defines a function, called f' , in terms of x . That function $y = f'(x)$ is called the derivative function of f or in short it is called

the derivative of f .

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists

Domain of $f' = \{x \mid f'(x) \text{ exists}\}$

E.g. $f(x) = x^2$


$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$\frac{x^2 - x^2}{h} = \frac{0}{0}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} (2x + h) = 2x$$


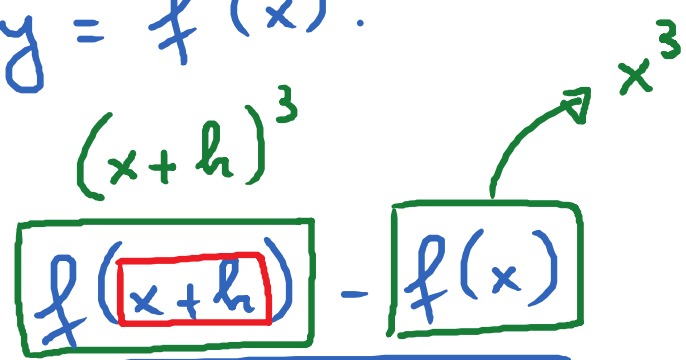
$f'(x) = 2x \rightarrow$ the derivative of the function $f(x) = x^2$ is given by the formula $f'(x) = 2x$

So, $f'(1) = 2$; $f'(2) = 4$; $f'(\frac{7}{2}) = 7$

$$f'(\frac{9}{4}) = \frac{9}{2} \dots$$

Now, consider $f(x) = x^3$. We want to find the formula for $y = f'(x)$.

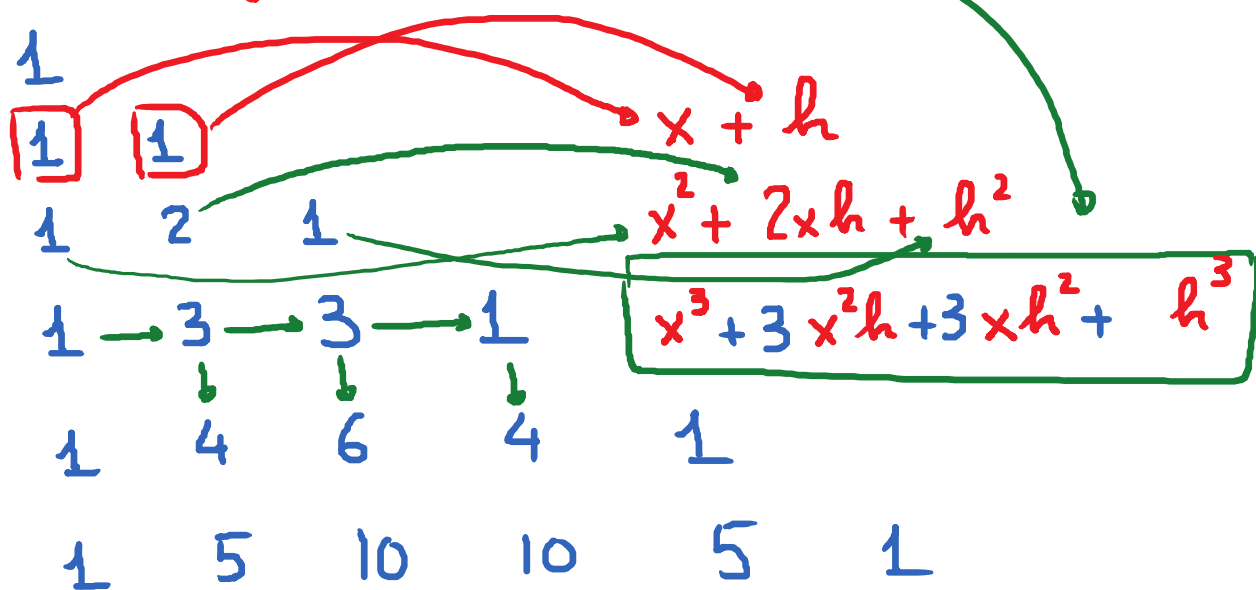
By definition,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$


$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$\left(\frac{0}{0} \right)$$

Pascal Triangle:



$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = 3x^2.$$

If $f(x) = x^3$, then $f'(x) = 3x^2$.

E.g. Equation of the tangent line to graph
of $f(x) = x^3$ at the point where $x = -2$

$$\text{Slope} = f'(-2) = 3 \cdot (-2)^2 = \boxed{12}$$

Point $(-2, -8)$.

$$\text{Equation: } y + 8 = 12(x + 2)$$

$$y = 12x + 16$$

So far, function $f(x)$	derivative $f'(x)$
$f(x) = x^2$	$f'(x) = 2x$
$f(x) = x^3$	$f'(x) = 3x^2$
$f(x) = x^4$	$f'(x) = 4x^3$
$f(x) = x^{2018}$	$f'(x) = 2018x^{2017}$

E.g. $f(x) = \sqrt{x}$

Use the definition of the derivative function to develop the formula for $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{\overbrace{f(x+h)}^{\sqrt{x+h}} - \underbrace{f(x)}_{\sqrt{x}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x+h} - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

If $f(x) = \sqrt{x}$, then $f'(x) = \frac{1}{2\sqrt{x}}$.

Find $f'(100) = \frac{1}{20}$; $f'(81) = \frac{1}{18}$

$f'(0) = \text{DNE}$; $f'(-2) = \text{DNE}$.

Def: We say that a function f is differentiable at a point x if $f'(x)$ exists, i.e., the derivative exists at that point.