For $f(x)=x^2$, $f'(x)=2x$, it exists at every number in $(-\infty,\infty)$. So, the function $f(x)=x^2$ is differentiable everywhere.

For $f(x)=\sqrt{x}$, $f'(x)=\dfrac{1}{2\sqrt{x}}$, it exists at every number in $(0,\infty)$.

So, $f(x)=\sqrt{x}$ is differentiable on $(0,\infty)$.

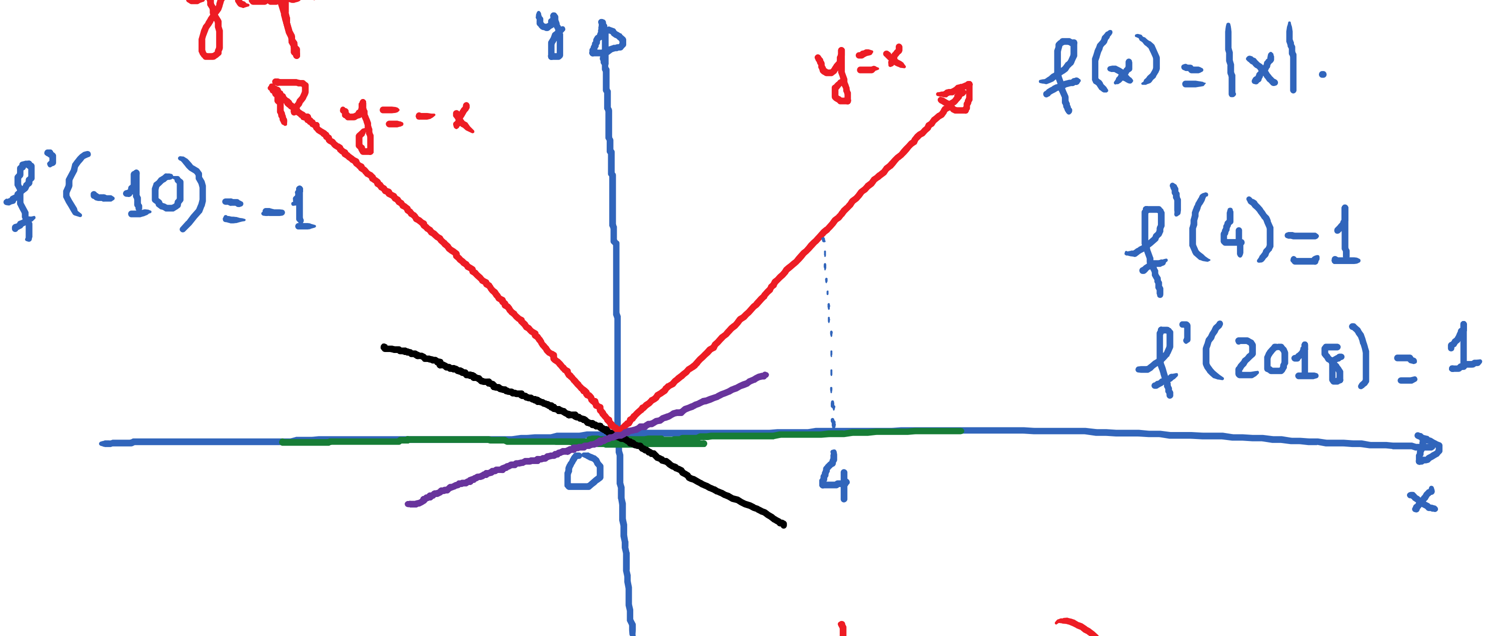
$f(x)=\sqrt{x}$ is NOT differentiable on $(-\infty,0]$.

$f(x)$	$f'(x)$
x^2	$2x$
x^3	$3x^2$
x^{2018}	$2018x^{2017}$
$\sqrt{x}$	$\dfrac{1}{2\sqrt{x}}$

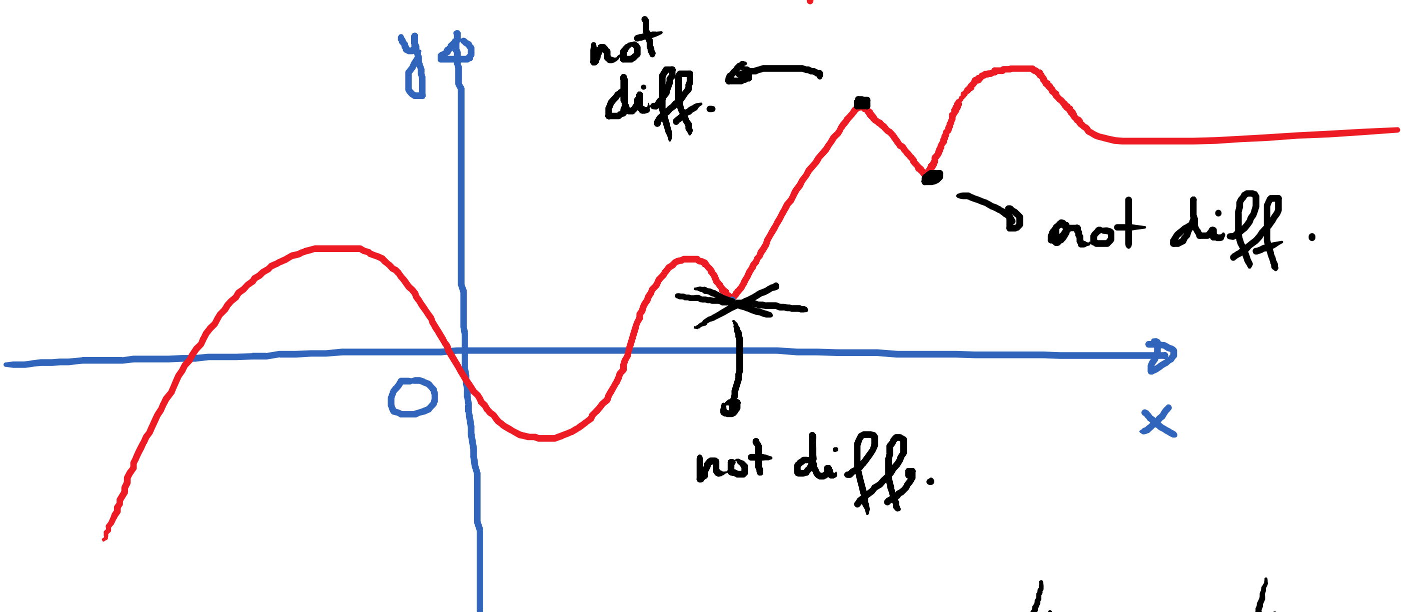
$x^{\frac{1}{2}} \to \frac{1}{2}x^{\frac{1}{2}-1}$

$\frac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2x^{\frac{1}{2}}}$

Q: How to detect differentiability from a graph?



$f'(0)$ DNE.



$$\frac{\frac{4}{3.1} - \frac{4}{3}}{0.1}$$

E.g. Use definition of the derivative to show that for the function $f(x) = |x|$, the derivative does not exist at $x = 0$; i.e., $f'(0)$ DNE.

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

0

x

$$= \lim_{h \to 0} \frac{f(h) - \boxed{f(0)}}{h} \qquad |0| = 0$$

$$= \lim_{h \to 0} \frac{|h|}{h}$$

$$\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} (1) = 1$$

0

$h > 0$

$$\lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0} \left(\frac{-h}{h} \right) = \lim_{h \to 0} (-1) = -1$$

0

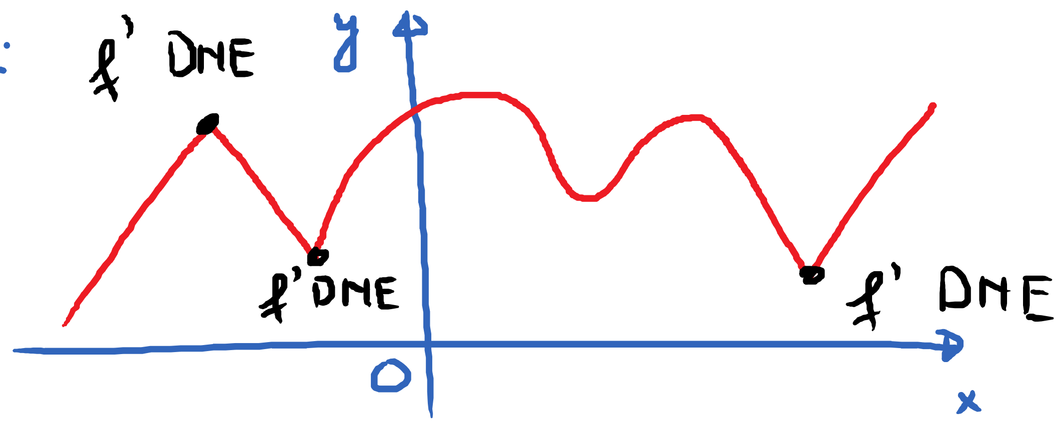
$h < 0$

$\neq$

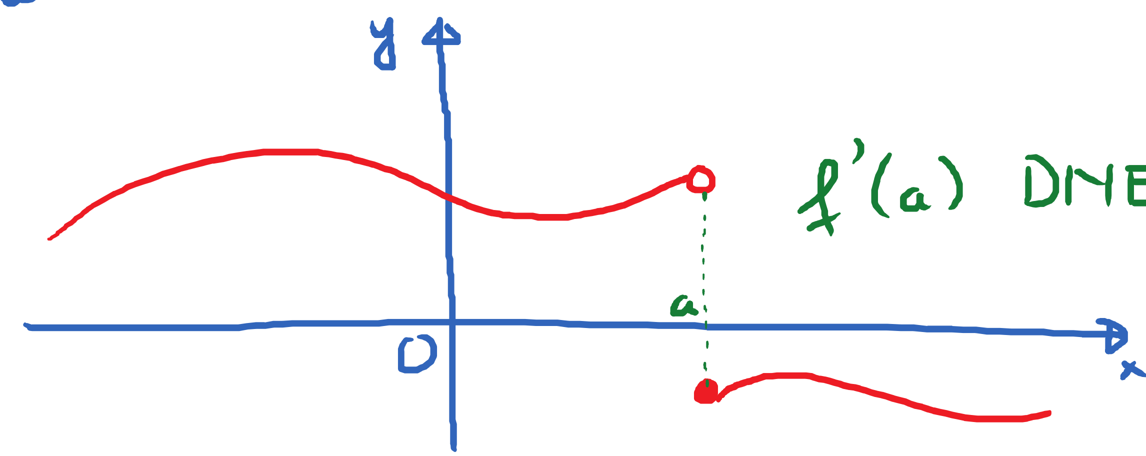
Since left limit $\neq$ right limit, limit DNE. So, $f'(0)$ DNE.

In general, if a graph has a "corner" at a point, the derivative DNE there.

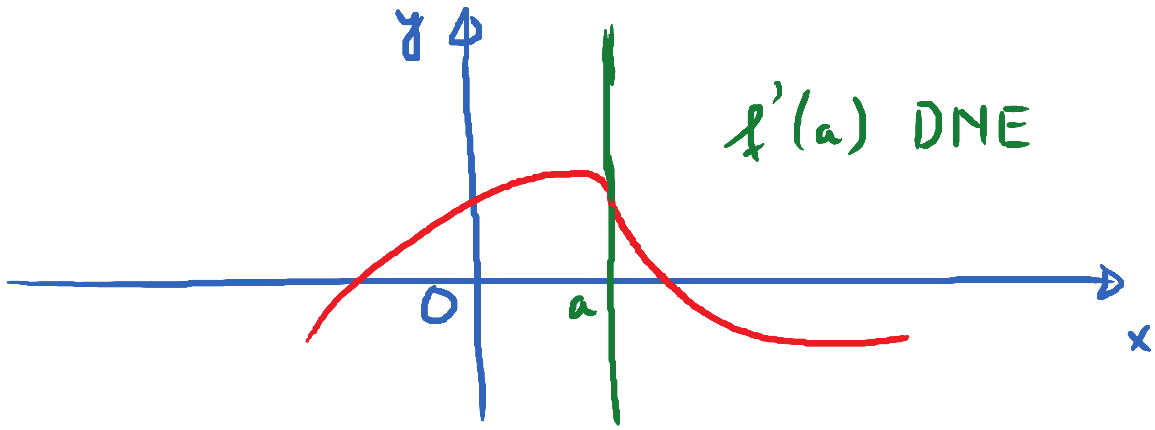
E.g.



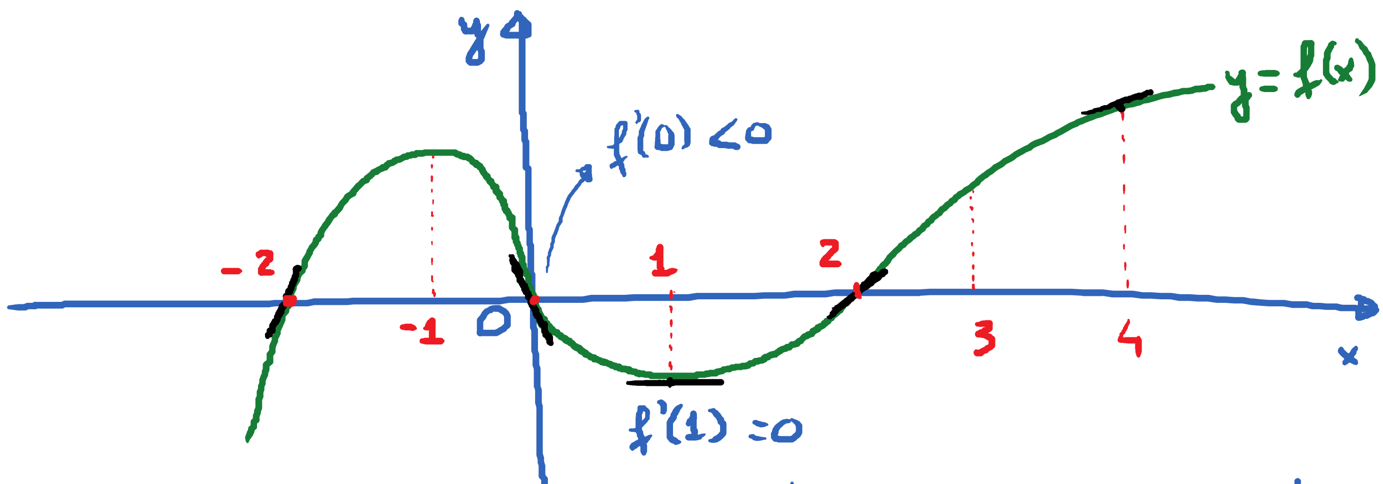
Also, if f is NOT continuous a point, f' DNE there



If f has a vertical tangent line at a point, f' DNE either.



E.g.



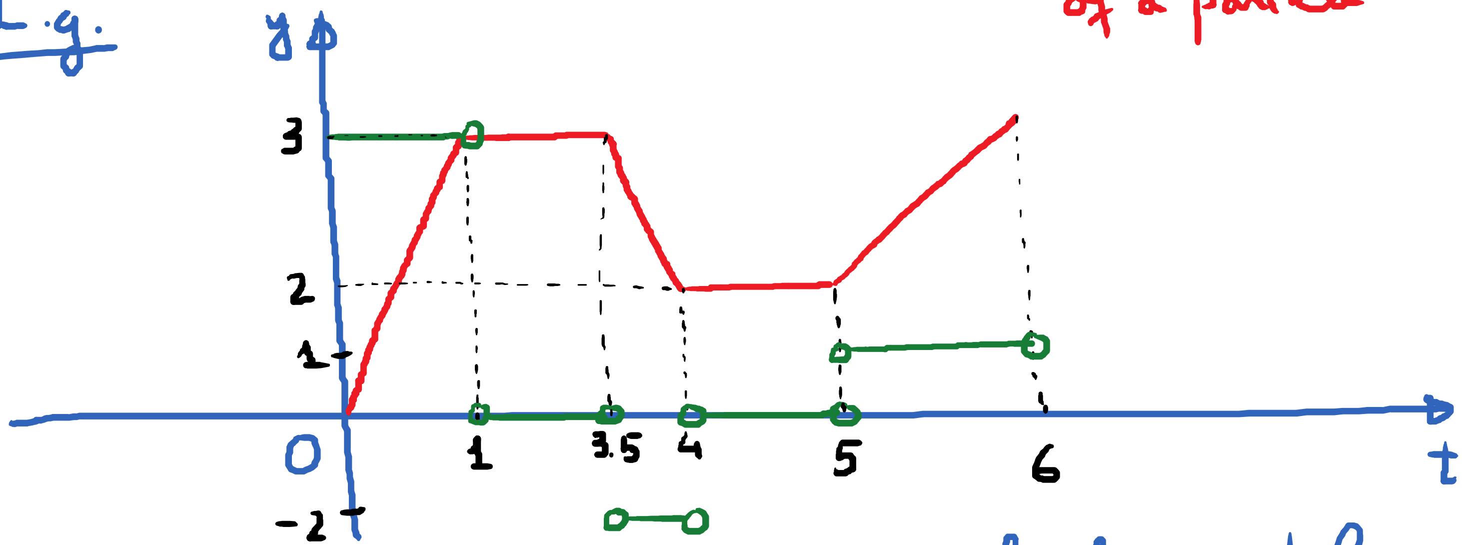
Q: Arrange the following numbers in increasing order:

$f'(-2)$, $f'(0)$, $f'(2)$, $f'(4)$; $f'(1)$

$f'(0) < f'(1) < f'(4) < f'(2) < f'(-2)$

$y = f(t)$: position function of a particle

E.g.



Q: Graph the velocity function of this particle.