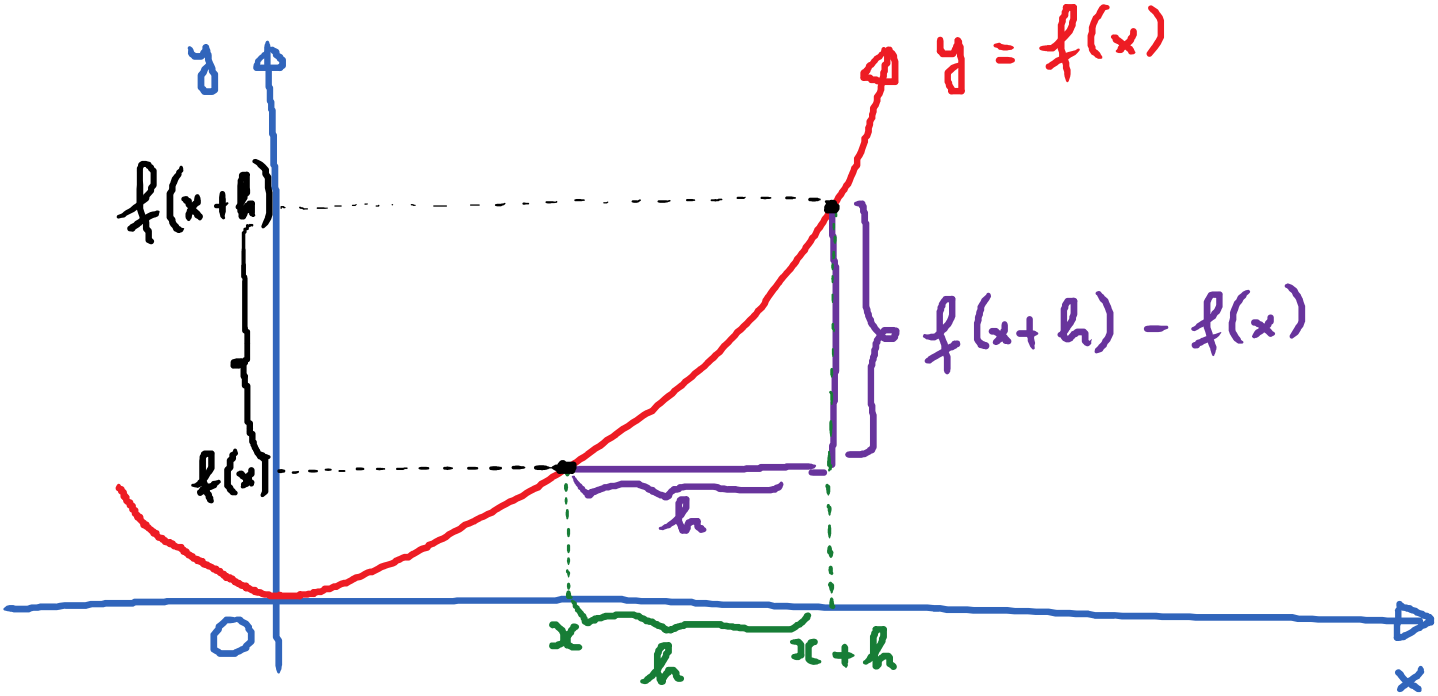
3.4. Derivatives and Rates of Change

Thursday, July 19, 2018 7:32 AM

Recall:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



$$f'(x) = \lim_{\text{Change in } x \to 0} \frac{\text{Change in } y}{\text{Change in } x} = \lim_{\Delta x \to 0} \boxed{\frac{\Delta y}{\Delta x}}$$

rate of change of $y = f(x)$

$$\rightarrow \boxed{f'(x) = \text{instantaneous R.O.C. of } f \text{ at } x}$$

In physics, $y = f(t)$ = position function of an object

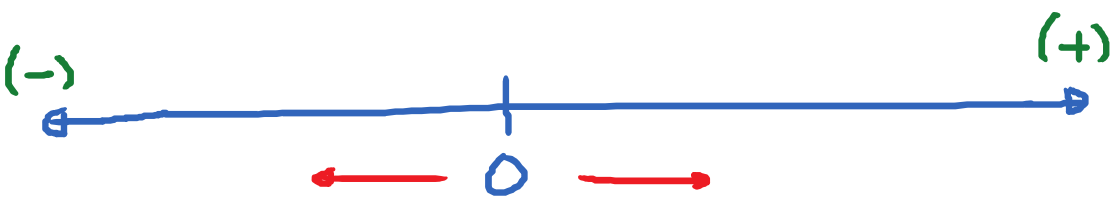
Then $v(t) = f'(t)$ = Instantaneous R.O.C. of position

= Instantaneous velocity

Then $a(t) = f''(t)$ = Instantaneous acceleration.

E.g. The position function of a particle moving along an axis is given by the function.

$$s(t) = t^3 - 9t^2 + 24t + 4 \; ; \; t \geq 0$$



ⓐ At what time(s) is the particle at rest?

means $v(t) = 0$

Find $v(t)$ and set it equal to 0.

$$v(t) = s'(t) = 3t^2 - 18t + 24$$

$$3t^2 - 18t + 24 = 0$$

$$3(t^2 - 6t + 8) = 0$$

$$\boxed{3(t-2)(t-4)} = 0$$

$\longrightarrow t = 2$; $t = 4$.

b During which time interval(s) is the particle moving from left to right; i.e., move in the (+) direction. / moving from right to left; i.e., move in the (−) direction.

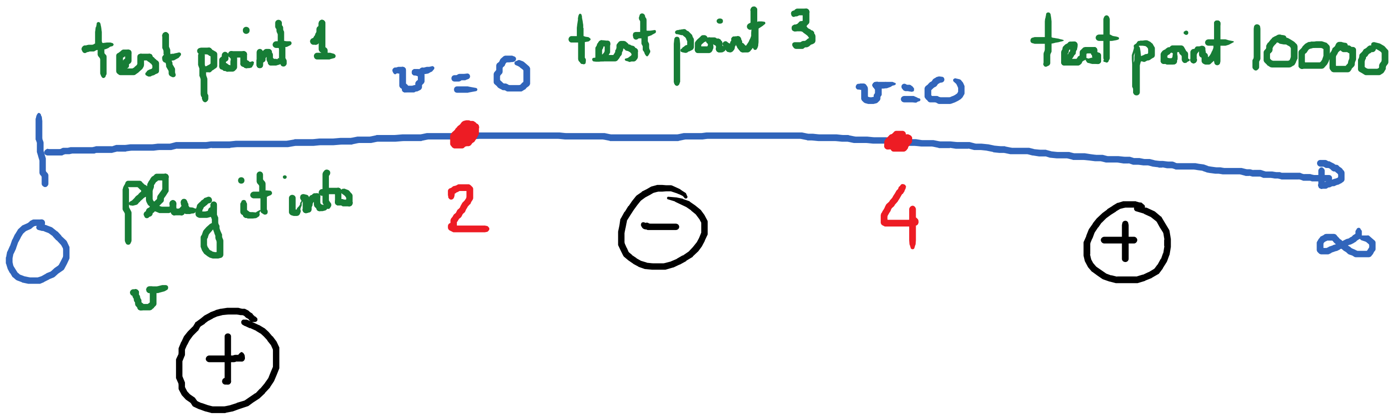
Want: Find time interval(s) on which

$v(t) > 0$ (move in the (+))

and

$v(t) < 0$ (move in the (−))

Important strategy:



Conclusion: $v(t) > 0$ on $(0,2) \cup (4,\infty)$

$v(t) < 0$ on $(2,4)$

Particle moves from left to right on $(0,2) \cup (4,\infty)$

________ right to left on $(2,4)$.

(c) During which time interval(s) is the particle speeding up / slowing down.

what do these mean in terms of the functions that describe the movement of the particle?