Thursday, July 19, 2018 8:00 AM

Note: Speeding up $\left[\begin{array}{l} a(t) > 0 \\ v(t) > 0 \end{array}\right.$ or $\left[\begin{array}{l} a(t) < 0 \\ v(t) < 0 \end{array}\right.$

bottom line: speeding up $\equiv$ $a(t)$ and $v(t)$ have the same sign.

Slowing down $\left[\begin{array}{l} a(t) > 0 \\ v(t) < 0 \end{array}\right.$ or $\left[\begin{array}{l} a(t) < 0 \\ v(t) > 0 \end{array}\right.$

Slowing down $\equiv$ $a(t)$ and $v(t)$ have opposite signs.

$\longrightarrow$ Problem becomes: finding the interval(s) on which $a(t)$ and $v(t)$ have the same sign / opposite signs

$$a(t) = s''(t) = v'(t) = 6t - 18$$

$$a(t) = 0 \longrightarrow t = 3$$

Table (of signs of a(t) and v(t))

t	0		2		3		4		∞
$v(t)$		$+$	0	$-$		$-$	0	$+$	
$a(t)$		$-$		$-$	0	$+$		$+$	

Conclusion:

Speed up : $(2,3) \cup (4,\infty)$

Slow down : $(0,2) \cup (3,4)$

Ex. $s(t) = \dfrac{t}{1+t^2}$, $t \geq 0$.

Time interval(s) on which object is speeding up or slowing down?

$$v(t) = s'(t) = \frac{1\cdot(1+t^2) - 2t\cdot t}{(1+t^2)^2}$$

$$v(t) = \frac{1+t^2-2t^2}{(1+t^2)^2} = \frac{1-t^2}{(1+t^2)^2} = \frac{1-t^2}{t^4+2t^2+1}$$

$$a(t) = v'(t) = \frac{-2t(t^4+2t^2+1) - (1-t^2)(4t^3+4t)}{(t^4+2t^2+1)^2}$$

$$= \frac{-2t^5 - 4t^3 - 2t - \cancel{4t^3} - 4t + 4t^5 + \cancel{4t^3}}{(t^4+2t^2+1)^2}$$

$a(t)$

$$= \frac{2t^5 - 4t^3 - 6t}{(t^4+2t^2+1)^2} = \boxed{\frac{2t(t^4-2t^2-3)}{(t^4+2t^2+1)^2}}$$

(b) Slow down / Speed up.

Step 1: $v(t) = 0$. ; $a(t) = 0$

$$v(t) = \frac{1-t^2}{(1+t^2)^2} = 0 \longleftrightarrow 1 - t^2 = 0$$

$$t^2 = 1$$

$$t = \pm 1$$

Since $t \geq 0$, we choose $t = 1$

$$a(t) = \frac{2t(t^4 - 2t^2 - 3)}{(t^4 + 2t^2 + 1)^2} = 0$$

$$\Longleftrightarrow \quad 2t(t^4 - 2t^2 - 3) = 0$$

Either $2t = 0$ or $t^4 - 2t^2 - 3 = 0$

$\underbrace{\boxed{t = 0}}$ $\qquad (t^2 - 3)(t^2 + 1) = 0$

$t^2 - 3 = 0$ or $t^2 + 1 = 0$

$t^2 = 3 \qquad \cancel{t^2 = -1}$

$t = \pm\sqrt{3}$

$t \geq 0$; $\boxed{t = \sqrt{3}}$

t	0	1	$\sqrt{3}$	∞
$v(t)$		0		
$a(t)$			0	

Note for HW: $y = C(x) \rightarrow$ cost function.

Marginal cost $= C'(x)$