3.5 Derivatives of Trig Functions

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Recall: The differentiation rules that we developed so far:

$$\frac{d}{dx}\left[x^n\right] = n x^{n-1} \quad \text{(Power Rule)}$$

u, v : functions of x, k : constant

$$\frac{d}{dx}\left[k \cdot u\right] = k \cdot \frac{du}{dx} \quad \text{(Constant multiple rule)}$$

$$\frac{d}{dx}\left[u \pm v\right] = \frac{du}{dx} \pm \frac{dv}{dx} \quad \text{(Sum/difference rule)}$$

$$\frac{d}{dx}\left[u \cdot v\right] = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx} \quad \text{(product rule)}$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2} \quad \text{(quotient rule)}$$

→ Develop formulas for the derivatives of functions like $\sin(x)$, $\cos(x)$, $\tan(x)$, $\cot(x)$, $\csc(x)$, $\sec(x)$

Reminder of basic trig identities.

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \quad ; \quad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)} \quad ; \quad \csc(x) = \frac{1}{\sin(x)}$$

α, β : angles.

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\sin^2(x) + \cos^2(x) = 1$$

Goal: Develop the formula for the derivative of $f(x) = \sin(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{h \to \boxed{0}} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{\sin(x)} + \cos(x)\sin(h) \cancel{- \sin(x)}}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x) \cdot \sin(h)}{h}$$

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$$= \cos(x) \cdot \boxed{\lim_{h \to 0} \frac{\sin(h)}{h}} = \cos(x)$$

If $f(x) = \sin(x)$, then $f'(x) = \cos(x)$.

In Leibnitz notation:

$$\boxed{\frac{d}{dx}\left[\sin(x)\right] = \cos(x)}$$

In a similar way, we can obtain:

$$\boxed{\frac{d}{dx}\left[\cos(x)\right] = -\sin(x)}$$

Develop formula for $\frac{d}{dx}\left[\tan(x)\right]$.

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\left[\tan(x)\right]' = \frac{\cos(x)\cdot\cos(x) - \sin(x)\left(-\sin(x)\right)}{\cos^2(x)}$$

$$= \frac{\underbrace{\boxed{\cos^2(x) + \sin^2(x)}}_{}^{\to 1}}{\cos^2(x)}$$

$$[\tan(x)]' = \frac{1}{\cos^2(x)} = \sec^2(x).$$

$$\boxed{\frac{d}{dx}[\tan(x)] = \sec^2(x)}$$

Ex. Develop the formulas for:

$$\frac{d}{dx}[\cot(x)] ; \quad \frac{d}{dx}[\sec(x)]$$

$$\frac{d}{dx}[\csc(x)]$$

$$\boxed{\frac{d}{dx}[\cot(x)] = -\csc^2(x)}$$

$$\boxed{\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)}$$

$$\boxed{\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)}$$

Ex let $y = x^2 \sin(x)$.

Find $\frac{dy}{dx}$?

product rule

$$\frac{d}{dx}[\underbrace{x^2}_{u} \cdot \underbrace{\sin(x)}_{v}] = 2x \cdot \sin(x) + x^2 \cdot \cos(x)$$

Higher order derivatives of $y = \sin x$ and $y = \cos x$.

$$y = \sin(x) \quad ; \quad \frac{dy}{dx} = \cos(x)$$

$$\frac{d^2y}{dx^2} = -\sin(x) \quad ; \quad \frac{d^3y}{dx^3} = -\cos(x)$$

$$\frac{d^4y}{dx^4} = \sin(x).$$

2018 divided by 4

→ Remainder = 2

$$\frac{d^{2018}y}{dx^{2018}} = -\sin(x)$$

$$\frac{d^ny}{dx^n} = \begin{cases} 4\lfloor n \quad R=0 \longrightarrow \sin(x) \\ 4\lfloor n \quad R=1 \longrightarrow \cos(x) \\ 4\lfloor n \quad R=2 \longrightarrow -\sin(x) \\ 4\lfloor n \quad R=3 \longrightarrow -\cos(x) \end{cases}$$

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If $y = \cos(x)$, then

$$\frac{dy^n}{dx^n} \begin{cases} 4\,\underline{|\,n} \quad R=0 \longrightarrow \cos(x) \\ 4\,\underline{|\,n} \quad R=1 \longrightarrow -\sin(x) \\ 4\,\underline{|\,n} \quad R=2 \longrightarrow -\cos(x) \\ 4\,\underline{|\,n} \quad R=3 \longrightarrow \sin(x) \end{cases}$$