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Fact: Any rational function is continuous at every point within its domain.

E.g. $f(x) = \dfrac{2x+3}{x-5}$

Domain: $(-\infty, 5) \cup (5, \infty)$

or $\{x \mid x \neq 5\}$

f is continuous at any point x with $x \neq 5$.

f is not continuous at $x = 5$ b/c it violates

1

Fact: Any function in this class will be continuous on its domain.

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E.g. $f(x) = \sqrt{\boxed{x - 5}}$.

Find the interval within which f is continuous? $\longrightarrow$ just need to find domain

To find domain: $x - 5 \geq 0$

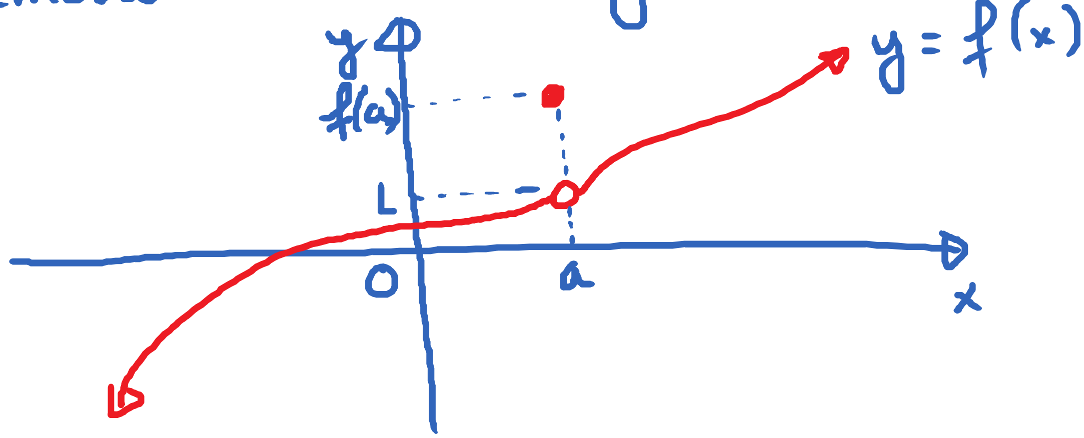
$\longrightarrow x \geq 5$

Domain: $[5, \infty)$

f is continuous at every point in $[5, \infty)$

2 Classify different types of discontinuity

I Removable discontinuity:



<u>Def</u>: f has a removable discontinuity at $x = a$

if (1) $\lim_{x \to a} f(x)$ exists; say, $\lim_{x \to a} f(x) = L$

(2) BUT $\lim_{x \to a} f(x) \neq f(a)$.

<u>E.g.</u> $f(x) = \begin{cases} \dfrac{x^2 + 3x + 2}{x + 2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2. \end{cases}$

<u>Claim</u>: f has a removable discontinuity at $x = -2$.

$$\lim_{x \to -2} \frac{x^2 + 3x + 2}{x + 2} \overset{\frac{0}{0}}{=} \lim_{x \to -2} \frac{\cancel{(x+2)}(x+1)}{\cancel{x+2}}$$

$$= \lim_{x \to -2} (x+1) = -1.$$

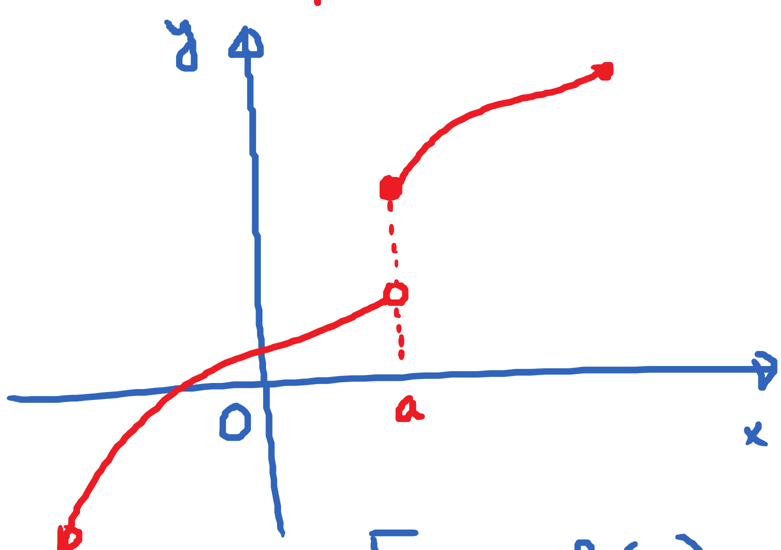
So, $\lim_{x \to -2} f(x)$ exists and equals -1

But $f(-2) = 1$.

So, $\lim_{x \to -2} f(x) \neq f(-2)$.

$\longrightarrow$ So, f has a removable discontinuity at $x = -2$.

II - Jump Discontinuity



<u>Def:</u> f has a jump discontinuity at $x = a$ if $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

<u>E.g.</u> $f(x) = \begin{cases} x\sin(x) & \text{if } x < \pi \\ x\cos(x) & \text{if } x \geq \pi. \end{cases}$

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f has a jump discontinuity at $x = \pi$.

<u>Reason:</u> $\lim\limits_{x \to \pi^-} f(x) = \lim\limits_{x \to \pi} (x \sin(x))$

$= \pi \cdot \sin(\pi) = 0$

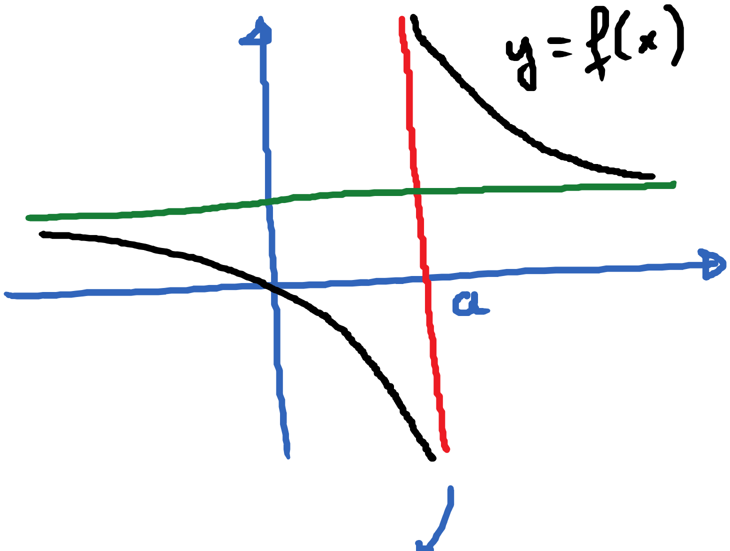
$\lim\limits_{x \to \pi^+} f(x) = \lim\limits_{x \to \pi} (x \cos(x))$

$= \pi \cdot \cos(\pi) = -\pi$.

$\neq$

So, left limit $\neq$ right limit.

III - Infinite Discontinuity.



V.A.

<u>Def:</u> f has an infinite discontinuity at $x = a$ if either

$\lim\limits_{x \to a^+} f(x) = \pm\infty$

OR $\lim\limits_{x \to a^-} f(x) = \pm\infty$