2.2. Functions and Graphs
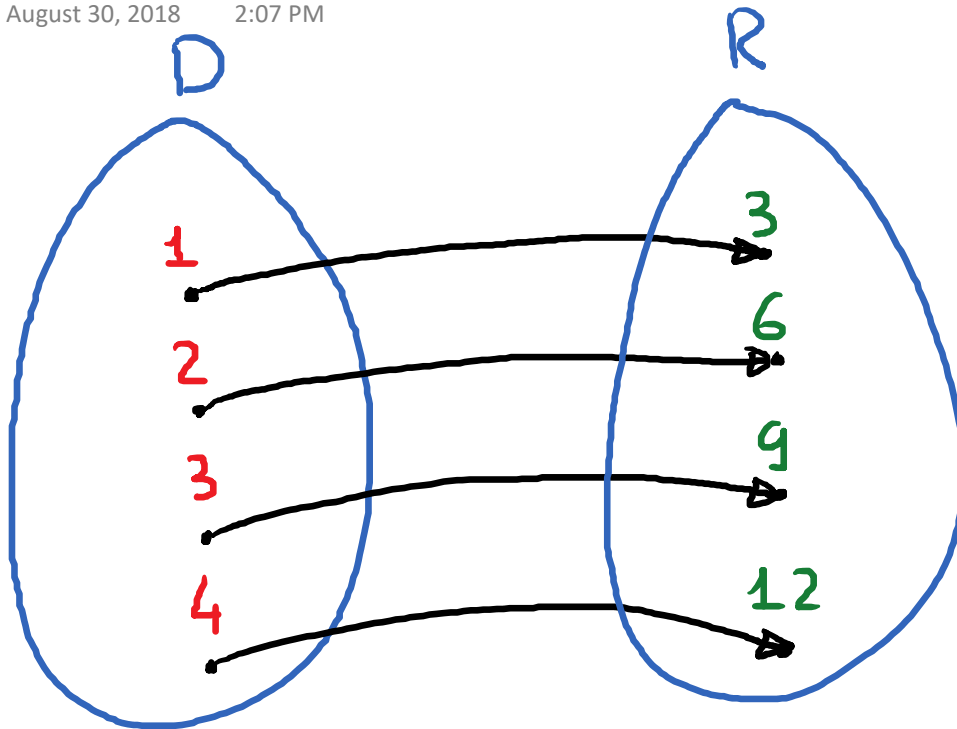
Objectives: ① Identifying Functions

② Finding Function Values
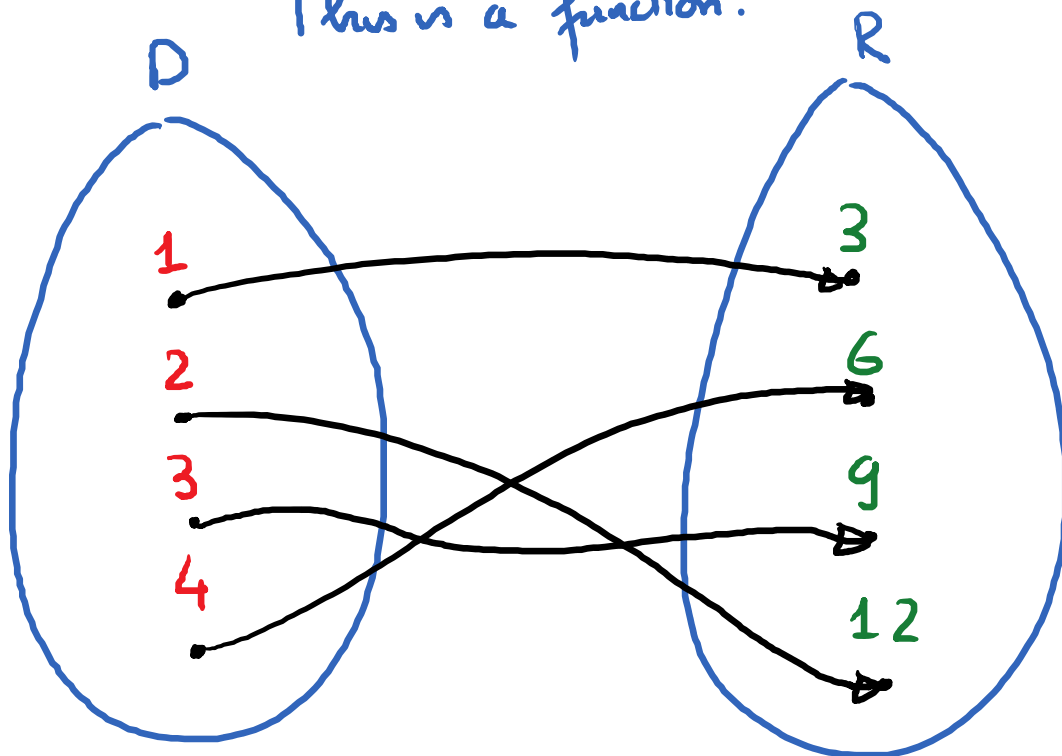
③ Graphs of Functions

④ The Vertical-Line Test

① Definition of a function:

A function is a ⟦correspondence⟧ between a first set called the ⟦domain⟧ and a second set called the ⟦range⟧ such that each member of the domain corresponds to ⟦exactly one⟧ member of the range.
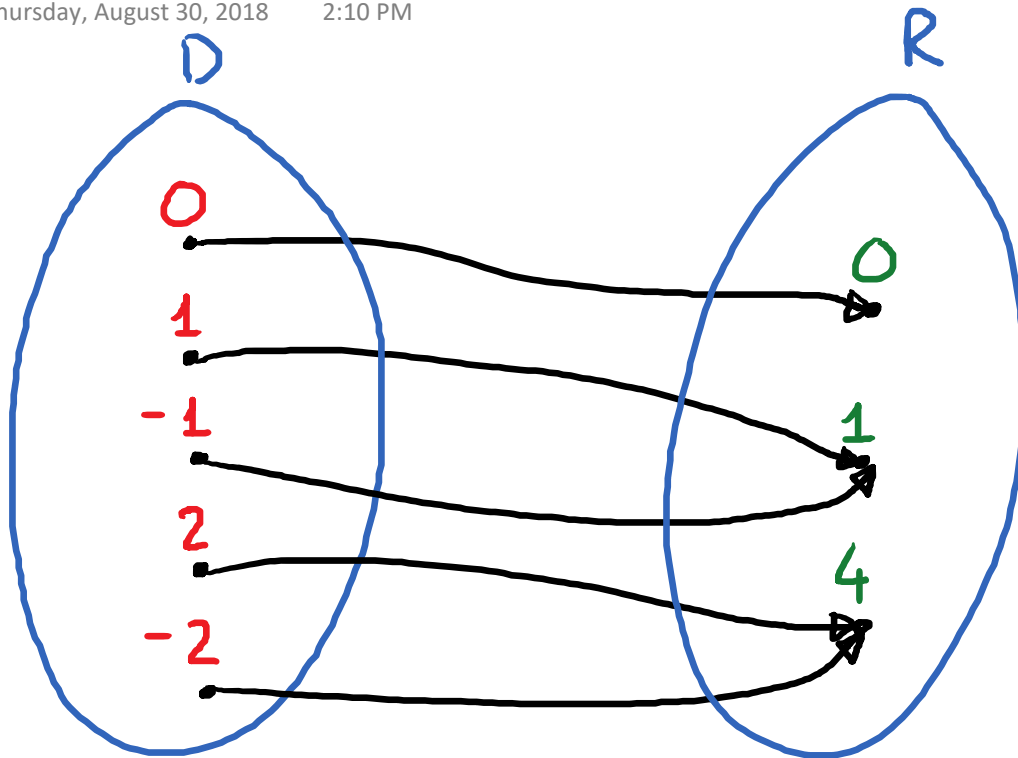
E.g.

D   R

1 ⟶ 3

2 ⟶ 6

3 ⟶ 9

4 ⟶ 12

This is a function.

D   R

1 ⟶ 3

2 ⟶ 12

3 ⟶ 9

4 ⟶ 6

This is a function

D   R

0 → 0
1 → 1
-1 → 1
2 → 4
-2 → 4

This is a function.

D   R

0 → 0
1 → 1
1 → -1
4 → 2
4 → -2
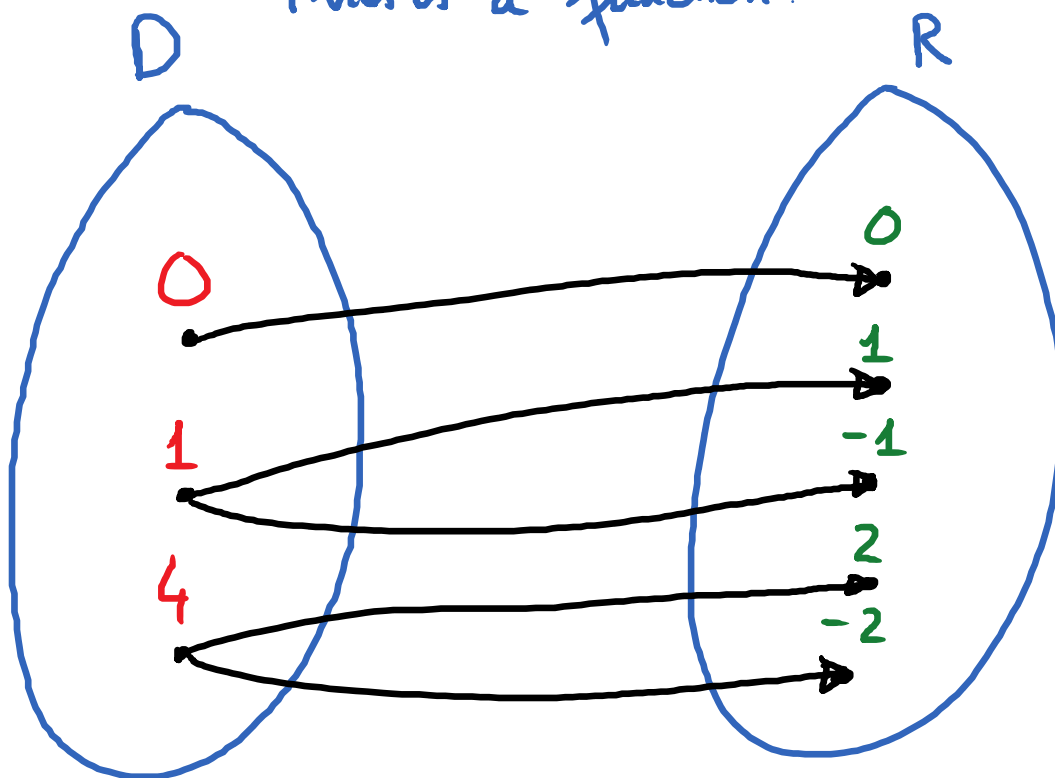
This is NOT a function.

E.g.   Domain : a set of numbers

Range : a set of numbers.

Correspondence: take a number in the domain and take the cube of it to obtain a number in the range

→ It is most convenient to describe this function as an equation.

function notation

$$y = x^3 \qquad \text{on} \qquad f(x) = x^3$$

this is read as f of x or the value of the function f at x

f is the name of the function

This equation describes the above correspondence.

② Evaluating functions (or finding values of functions)

Many functions are given to us as an equation.

For e.g. $f(x) = x^2$; $g(x) = \sqrt{x}$; $h(x) = \frac{1}{x}$;

etc.

E.g. $f(x) = \boxed{x}^2$

$f(\boxed{2}) = (2)^2 = 4$; $f(-2) = (-2)^2 = 4$

find the value in the
range that corresponds
to $x = 2$ in the domain

E.g. on finding function values.

$g(x) = 3 - 4\boxed{x}$ Find $g(\boxed{1}) = 3 - 4(1) = \boxed{-1}$

$g(-2) = 3 - 4 \cdot (-2) = \boxed{11}$

$3 - 4 \cdot (\boxed{})$

Find $g(b) = 3 - 4 \cdot (b) = \boxed{3 - 4b}$

$g(2b) = 3 - 4 \cdot (2b) = \boxed{3 - 8b}$

$g(b+1) = 3 - 4 \cdot (b+1)$

$\qquad = 3 - 4b - 4 = \boxed{-1 - 4b}$

$g(3x) = 3 - 4(3x) = \boxed{3 - 12x}$