2.4. Sum, Difference, Product, and Quotient of 2 functions

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Objectives: (1) The algebra of functions

(2) Find the domain of the sum, difference, product and quotient of 2 functions

(1) Suppose that f and g are functions of x.

The sum of f and g is a function is defined as:

$$(\underbrace{f+g}_{\text{name of the sum function}})(x) = f(x) + g(x)$$

E.g. $f(x) = 3 - x$; $g(x) = 3x^2 + 1$

$f(2) = 1$; $g(2) = 13$

$$(f+g)(2) = \underbrace{f(2)}_{1} + \underbrace{g(2)}_{13} = 1 + 13 = \boxed{14}$$

* Find the formula for the function $f+g$

$$(f+g)(x) = f(x) + g(x) = (3-x) + (3x^2+1)$$

$$\boxed{(f+g)(x) = 3x^2 - x + 4}$$

Note: We can find $(f+g)(2)$ by plugging $x=2$ to this formula:

$$(f+g)(2) = 3(2)^2 - 2 + 4 = \boxed{14}$$

E.g. Suppose that the functions f and g are given by the following graphs:

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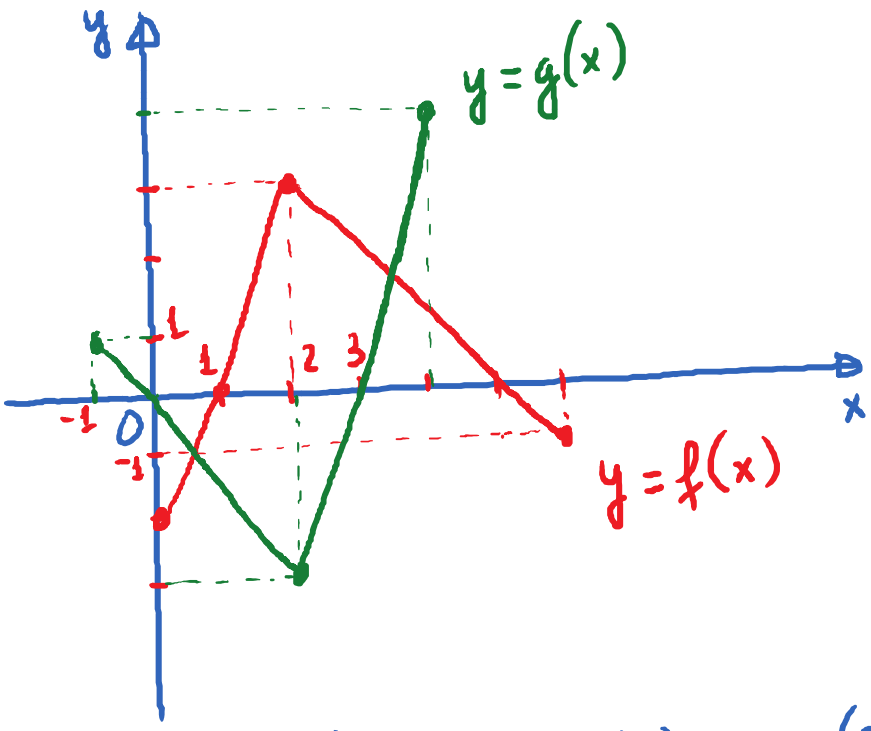

Q: Find $(f+g)(2)$?

Find $(f+g)(0)$?

$$(f+g)(2) = \underbrace{f(2)}_{3} + \underbrace{g(2)}_{-3} = 3 + (-3) = \boxed{0}$$

$$(f+g)(0) = \underbrace{f(0)}_{-2} + \underbrace{g(0)}_{0} = -2 + 0 = \boxed{-2}.$$

In a similar way, we can define the difference, the product and the quotient of 2 functions

$$(f-g)(x) = f(x) - g(x) \quad \text{(Difference)}$$

$$(f \cdot g)(x) = f(x) \cdot g(x) \quad \text{(Product)}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{(Quotient)}$$

provided that $g(x) \neq 0$

E.g. $f(x) = 3 - x$; $g(x) = 3x^2 + 1$.

Q1: Find $(f-g)(-2)$; $(f \cdot g)(-2)$; $\left(\frac{f}{g}\right)(-2)$?

Q2: Find the formula for

$(f-g)(x)$; $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$?

Q1: **Step 1:** Find $f(-2)$ and $g(-2)$

$f(-2) = 3-(-2) = 5.$

$g(-2) = 3(-2)^2 + 1 = 13$

Step 2: $(f-g)(-2) = \underbrace{f(-2)}_{5} - \underbrace{g(-2)}_{13} = 5 - 13 = \boxed{-8}.$

$(f \cdot g)(-2) = f(-2) \cdot g(-2) = (5)\cdot(13) = \boxed{65}.$

$\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \boxed{\frac{5}{13}}$

Q2: $*(f-g)(x) = f(x) - g(x)$

$= (3-x) - (3x^2+1)$

$= 3 - x - 3x^2 - 1$

$\boxed{(f-g)(x) = -3x^2 - x + 2}$

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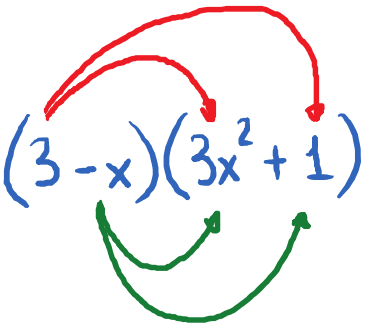
* $(f \cdot g)(x) = f(x) \cdot g(x) = (3-x)(3x^2+1)$



$= 9x^2 + 3 - 3x^3 - x$

$$(f \cdot g)(x) = -3x^3 + 9x^2 - x + 3$$

* $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3-x}{3x^2+1}$$

Ex. (1) $F(x) = x^2 - 6$; $G(x) = -3x$.

Find $(F-G)(-2)$

(2) $f(x) = 2x - 1$; $g(x) = x + 4$.

Find $(f \cdot g)(a)$

(3) $u(x) = 2 - x^2$; $v(x) = x - 1$. Find $\left(\frac{u}{v}\right)(3)$.

(1) $F(-2) = (-2)^2 - 6 = -2$; $G(-2) = (-3)(-2) = 6$

$(F-G)(-2) = -2-6 = \boxed{-8}$

(2) $f(a) = 2a-1$; $g(a) = a+4$

$(f \cdot g)(a) = (2a-1)(a+4) = 2a^2 + 8a - a - 4$

$\boxed{(f \cdot g)(a) = 2a^2 + 7a - 4}$

(3) $u(3) = 2-(3)^2 = -7$; $v(3) = 3-1 = 2$

$\left(\frac{u}{v}\right)(3) = \boxed{\frac{-7}{2}}$