2.4. Sum, Difference, Product and Quotient of 2 functions

Thursday, September 6, 2018 11:04 AM

Objectives: (1) The algebra of functions

(2) Find domain of the sum, difference, product, quotient of 2 functions

(1) Suppose that f and g are functions of x.

The sum of f and g is a function defined as:

$$(f+g)(x) = f(x) + g(x)$$

E.g. $f(x) = 3 - x$; $g(x) = 3x^2 + 1$.

$f(2) = 1$; $g(2) = 13$.

$$(f+g)(2) = f(2) + g(2) = 1 + 13 = 14$$

$$(f+g)(x) = f(x) + g(x)$$

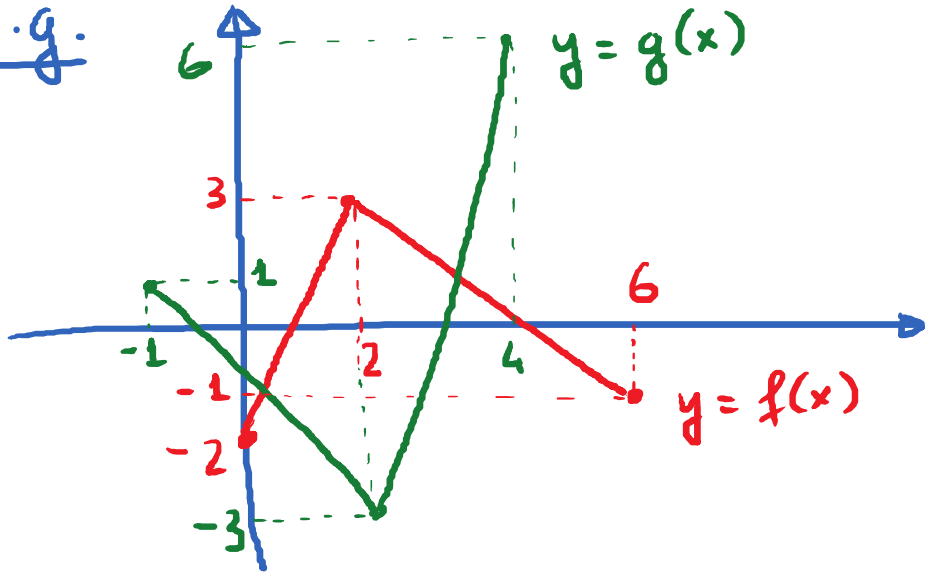
$$= (3 - x) + (3x^2 + 1)$$

$$(f+g)(x) = 3x^2 - x + 4$$

$$(f+g)(x) = 3x^2 - x + 4$$

Another way to find $(f+g)(2) = 3(2)^2 - 2 + 4 = \boxed{14}$

E.g.



Find $(f+g)(2)$?

$$(f+g)(2) = \underbrace{f(2)}_{3} + \underbrace{g(2)}_{-3} = 3 + (-3) = 0$$

In a similar way, we can define the difference, the product and the quotient of 2 functions.

$(f-g)(x) = f(x) - g(x)$ (Difference)

$(f \cdot g)(x) = f(x) \cdot g(x)$ (Product)

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (Quotient)

$\longrightarrow$ provided that $g(x) \neq 0$

E.g. $f(x) = 3 - x$; $g(x) = 3x^2 + 1$.

Q1: Find $(f-g)(-2)$; $(f \cdot g)(-2)$; $\left(\frac{f}{g}\right)(-2)$.

Q2: Find the formula for:

$(f-g)(x)$; $(f \cdot g)(x)$; $\left(\frac{f}{g}\right)(x)$.

Q1: Step 1: $f(-2) = 3 - (-2) = 5$

$g(-2) = 3(-2)^2 + 1 = 13$

Step 2: $(f-g)(-2) = f(-2) - g(-2)$

$= 5 - 13 = \boxed{-8}$

$$(f \cdot g)(-2) = f(-2) \cdot g(-2) = 5 \cdot 13 = \boxed{65}.$$

$$\left(\frac{f}{g}\right)(-2) = \frac{f(-2)}{g(-2)} = \boxed{\frac{5}{13}}$$

Q2: $(f-g)(x) = f(x) - g(x)$

$$= (3-x) - (3x^2+1)$$

$$= 3 - x - 3x^2 - 1$$

$$\boxed{(f-g)(x) = -3x^2 - x + 2}$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$= (3-x) \cdot (3x^2+1)$$

$$= 9x^2 + 3 - 3x^3 - x$$

$$\boxed{(f \cdot g)(x) = -3x^3 + 9x^2 - x + 3}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3-x}{3x^2+1}.$$

Ex. ① $F(x) = x^2 - 6$; $G(x) = -3x$.

Find $(F-G)(-2)$.

② $f(x) = 2x - 1$; $g(x) = x + 4$.

Find $(f \cdot g)(a)$

③ $u(x) = 2 - x^2$; $v(x) = x - 1$.

Find $\left(\frac{u}{v}\right)(3)$.

Sol: ① $F(-2) = (-2)^2 - 6 = 4 - 6 = -2$

$G(-2) = -3 \cdot (-2) = 6$

$(F-G)(-2) = F(-2) - G(-2)$

$= -2 - 6 = \boxed{-8}$.