E.g. A road rises 3 ft for every horizontal distance of 100 ft.

$$\text{Grade of Road} := \frac{\text{Vertical Change}}{\text{Horizontal Change}} = \frac{3}{100} = 0.03$$

Grade of Rd is 3%.

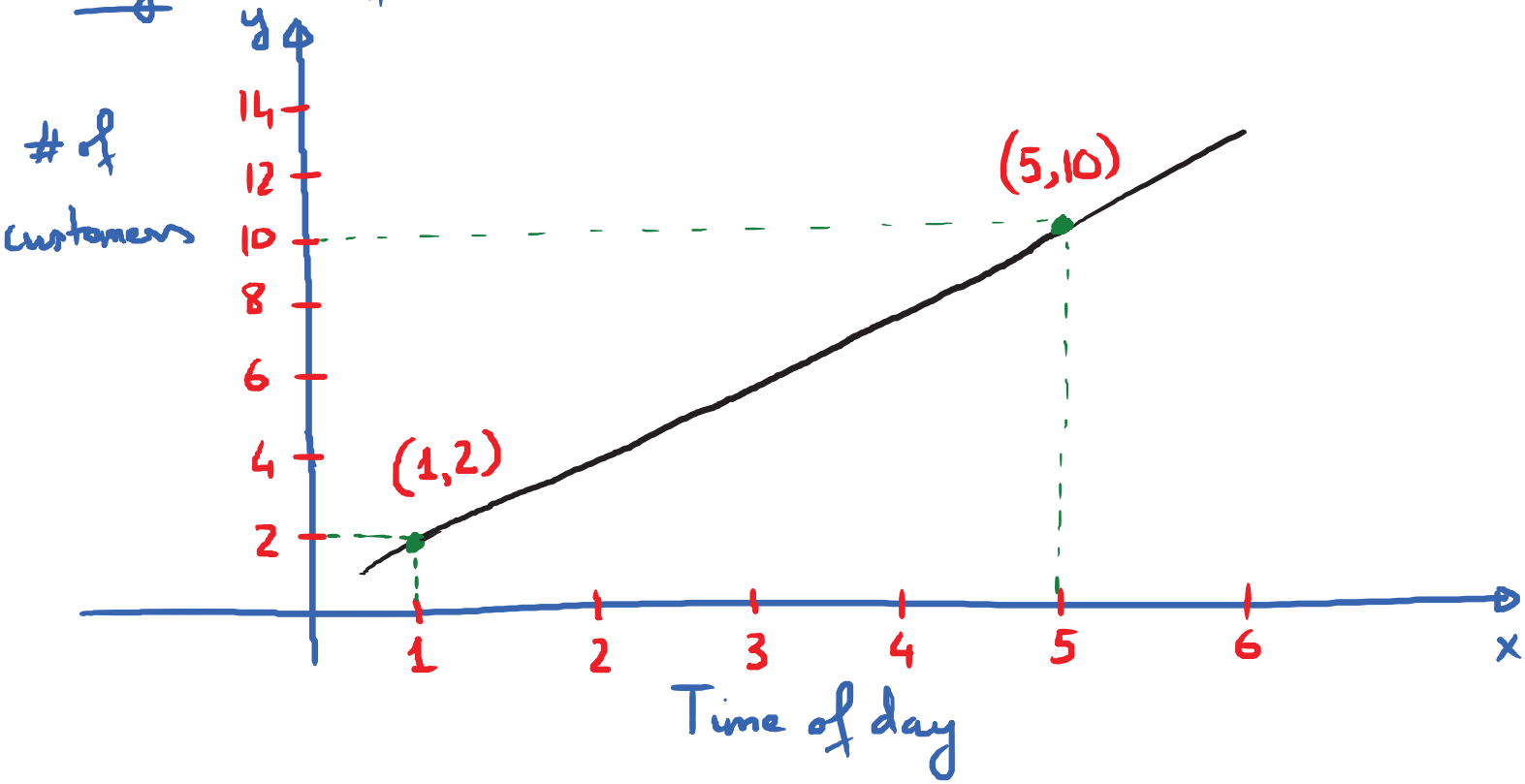
Ex. By 7 pm, Joe had typed 4 pages of his paper.
At 8:30pm, he had completed 10 pages.

Q: Find his typing rate in minutes per page.

* (4 pages, 7 pm) * (10 pages, 8:30 pm)

$$\text{Rate} = \frac{\text{Change in time}}{\text{Change in \# of pages}} = \frac{90 \text{ minutes}}{6 \text{ pages}} = 15 \text{ minutes/page}$$

E.g. Data from hair salon from a recent day of work



Q: Find the average # of hair cuts they do per hour?

$$\text{Rate} = \frac{\text{Change in \# of haircuts}}{\text{Change in time}} = \frac{10 \text{ haircuts} - 2 \text{ haircuts}}{5 \text{ hours} - 1 \text{ hours}}$$

$$= \frac{8 \text{ haircuts}}{4 \text{ hours}} = 2 \text{ haircuts per hour.}$$

④ Graph Linear Equations

* Using intercepts

E.g. $5x + 2y = 10$. Graph using intercepts.

Need: x-intercept and y-intercept.

x	y	
0	5	→ (0, 5) y-intercept
2	0	→ (2, 0) x-intercept

To find y-intercept, set $x = 0$:

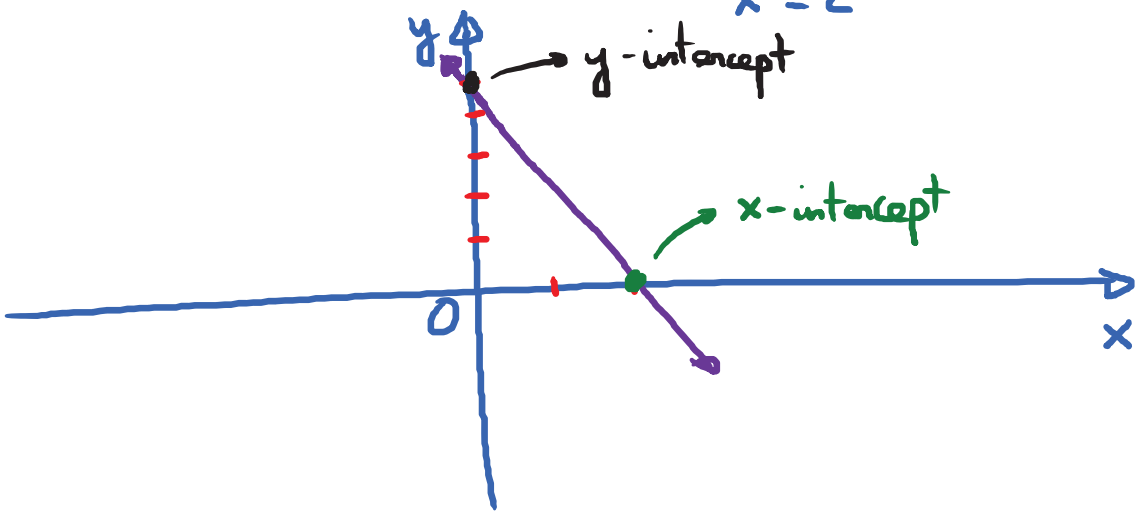
$5 \cdot (0) + 2y = 10$

$y = 5$

To find x-intercept, set $y = 0$:

$5x + 2 \cdot (0) = 10$

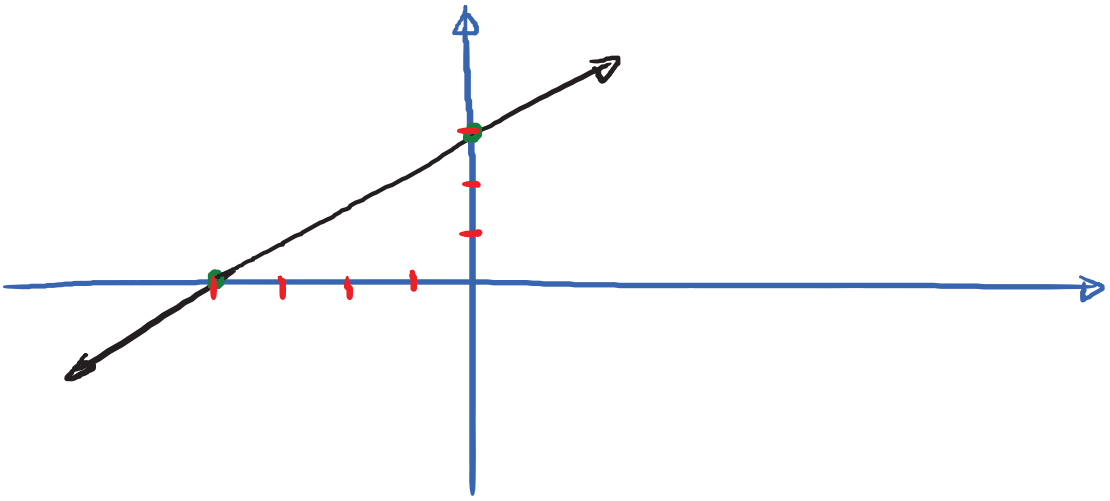
$x = 2$



E.g. Find the x-intercept and y-intercept of $3x - 4y = -12$ and use them to graph the line.

x-intercept : $(-4, 0)$

y-intercept : $(0, 3)$

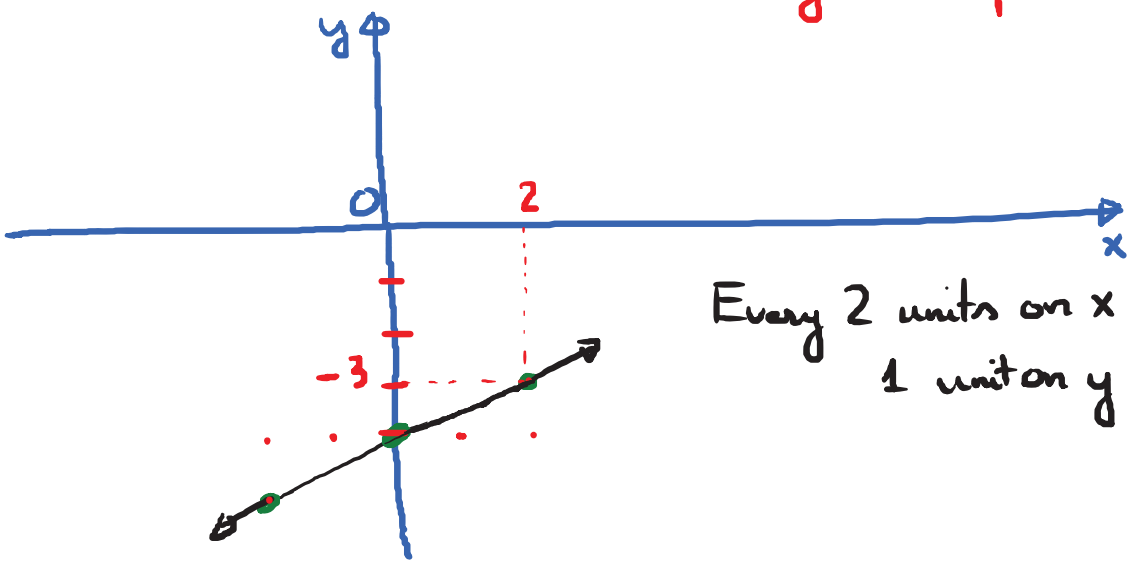


* Graph using slope and y-intercept

E.g. $y = \frac{1}{2}x - 4$

Slope $= \frac{1}{2}$

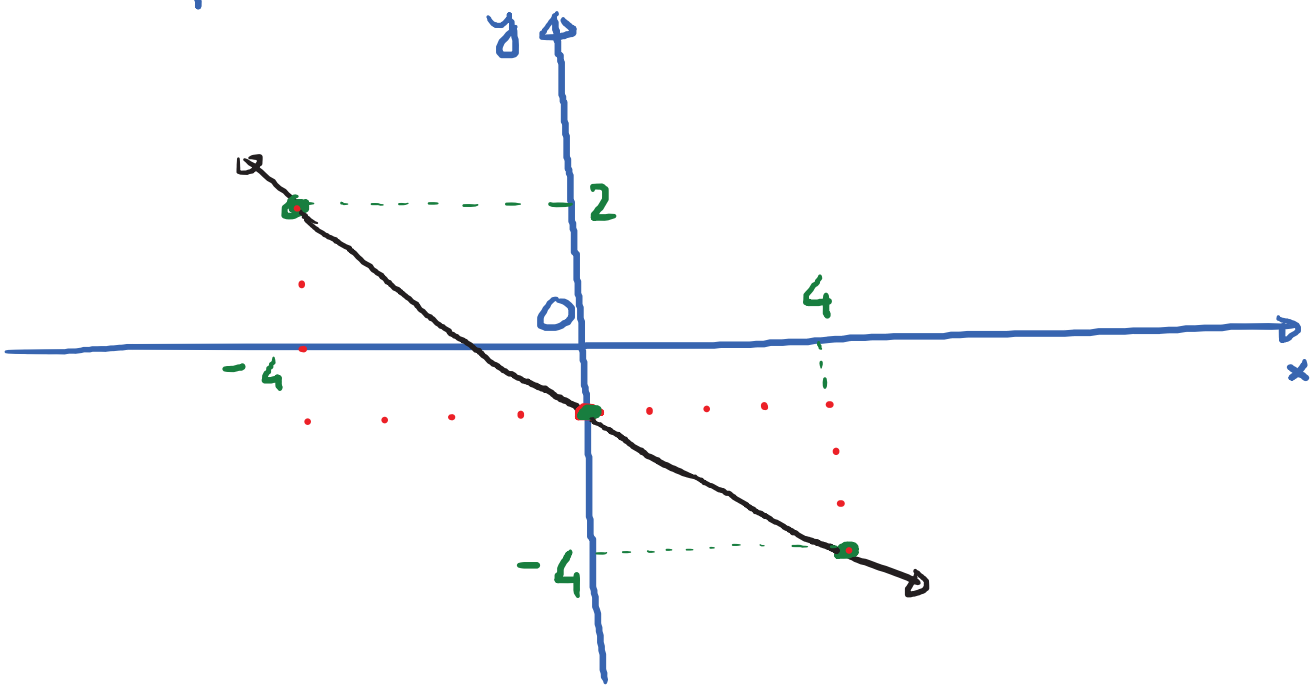
y-intercept : $(0, -4)$



Every 2 units on x corresponds to 1 unit on y b/c slope $= \frac{1}{2}$

Ex. $f(x) = -\frac{3}{4}x - 1$. Slope $= -\frac{3}{4}$; y-intercept: $(0, -1)$

Graph this using slope and y-intercept.

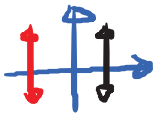


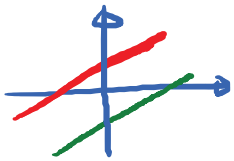
(5) Horizontal lines and Vertical lines

The graph of $y = b$ is a horizontal line with y-intercept $(0, b)$ and with slope $m = 0$.

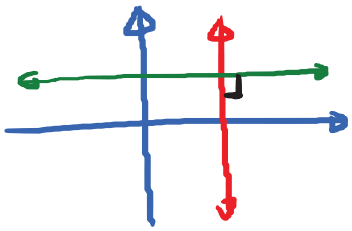
The graph of $x = a$ is a vertical line with x-intercept $(a, 0)$ and with slope undefined.

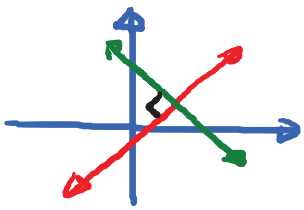
(6) Parallel and Perpendicular Lines

If 2 lines are vertical, they are parallel. 

For nonvertical lines, 2 lines are parallel if and only if they have the same slope. 

A vertical line and a horizontal line are perpendicular.



For nonvertical, nonhorizontal liners, 2 lines are perpendicular if and only if the product of their slopes is -1. In other words, one slope is the negative reciprocal of the other slope. 

E.g. $y = 2x - 5$; $2y - 4x = 3$

(L_1) (L_2)

Q: Are they parallel or perpendicular or neither?

(L_1): Slope $= 2$

(L_2): $2y - 4x = 3 \rightarrow 2y = 4x + 3 \rightarrow y = 2x + \frac{3}{2}$

Slope $= 2$

Answer: (L_1) and (L_2) are parallel b/c they have the same slope.

E.g. $5x - 6y = 30$; $5y + 6x = 0$

(L_1) (L_2)

Q: Are they parallel or perpendicular or neither?

(L_1): $-6y = -5x + 30 \rightarrow y = \boxed{\frac{5}{6}}x - 5$

(L_2): $5y = -6x \rightarrow y = \boxed{-\frac{6}{5}}x$

One slope is the neg. recip. of the other

$\rightarrow$ perpendicular