2.5 and 2.6. Linear Functions and Linear Equations

Tuesday, September 11, 2018 10:59 AM

Objectives: (1) Understand the equation $y = mx + b$ or $f(x) = mx + b$.

(2) Given 2 points on a line, find the slope of the line. Given a linear equation, derive the slope-intercept form of it.

(3) Solve some applications

(4) Graph linear functions
- using intercepts
- using slope and y-intercept

(5) Vertical lines and horizontal lines

(6) Parallel and Perpendicular lines

(1) A linear function is a function of the form

$y = mx + b$ or $f(x) = mx + b$

"Linear" is b/c the graph is a straight line.

m is the slope of the line.

$(0, b)$ is the y-intercept of the line.

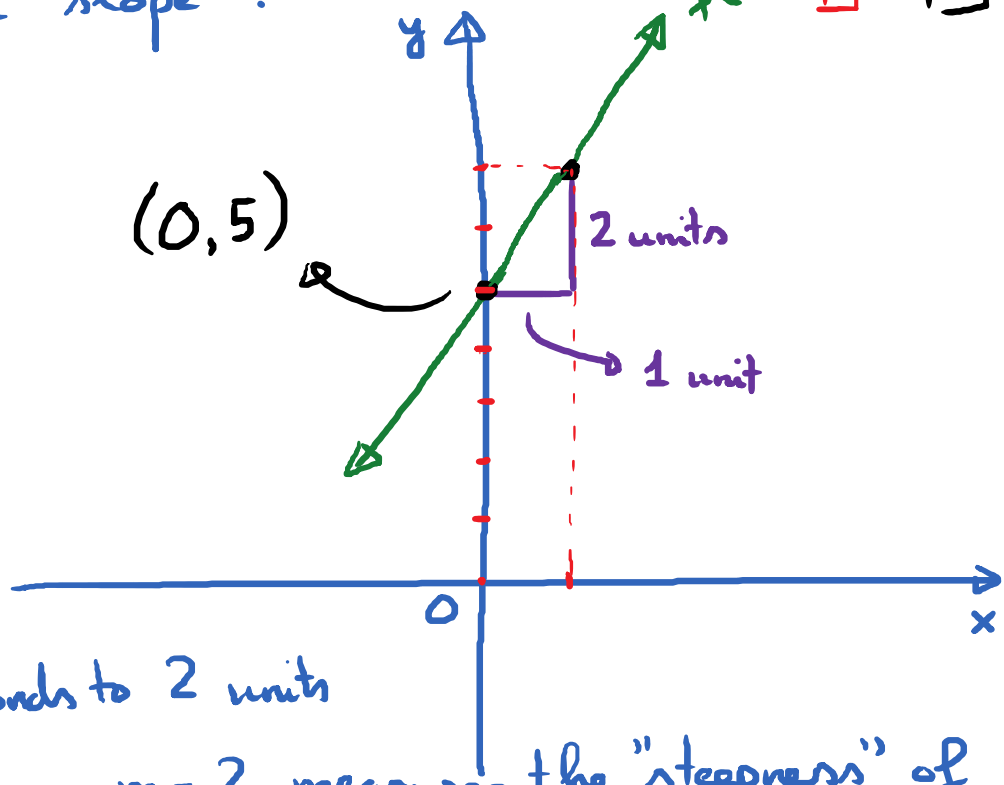
E.g. $f(x) = 2x + 5$. Here: $m = 2$; $b = 5$

slope

$f(x) = \boxed{2}x + \boxed{5}$

Why is m the "slope"?

x	$f(x) = 2x + 5$
0	5
1	7



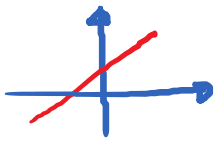
Every unit in the x-direction corresponds to 2 units in the y-direction $\rightarrow$ $m = 2$ measures the "steepness" of the line

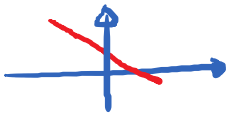
$(0, 5)$ is the intersection between graph and y-axis

$\rightarrow$ y-intercept.

To sum up, $f(x) = mx + b$

* m = slope → measures "steepness" of line.

$m > 0$: 

$m < 0$: 

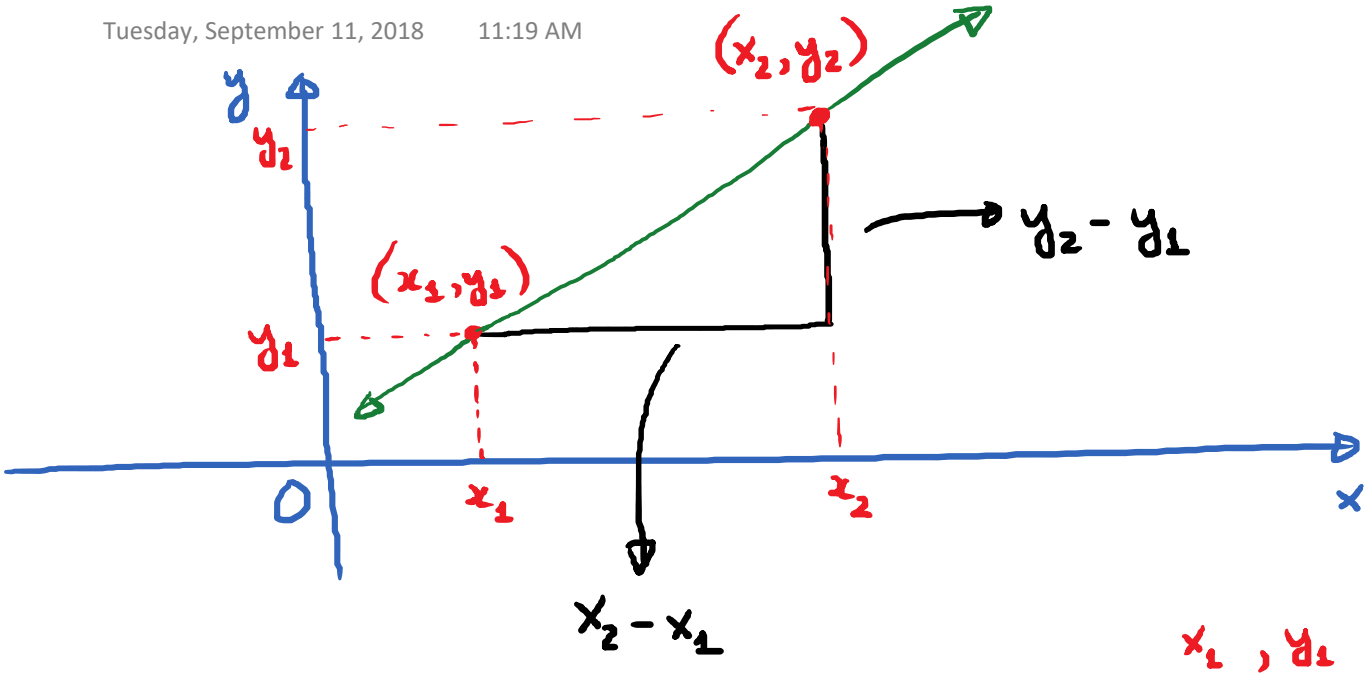
For every unit in the x-direction, the line rise/fall by $|m|$ units

* $(0, b)$ is the y-intercept.

(2) Calculate Slope

If (x_1, y_1) and (x_2, y_2) are points on a line L, then the slope of L is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{Rise}}{\text{Run}}$$



x_1 , y_1

E.g. Find the slope of the line containing $(9, -1)$ and $(-8, -7)$

x_2 y_2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - (-1)}{-8 - 9} = \frac{-6}{-17} = \frac{6}{17}$$

(y_2, y_1; x_2, x_1)

$$\approx 0.353$$

For every unit we move in the x-direction, the line rises by 0.353 unit.

Ex. Find the slope of the line containing $(7,-5)$ and $(3,2)$ and interpret the result.

$$m = \frac{2-(-5)}{3-7} = \frac{7}{-4} = -\frac{7}{4} = \boxed{-1.75.}$$

For every unit we moved in the x-direction, the line falls by 1.75 units

---

* $f(x) = mx + b$ is called the slope-intercept form.

Sometimes, we can be given linear equations that are not in this form.

→ Derive the slope-intercept form of any linear equation

E.g. $x + 2y = 8$.

Q: Find the slope-intercept form and find the slope and the y-intercept ?

Idea: Get y by itself.

$x + 2y = 8 \longrightarrow 2y = -x + 8 \longrightarrow y = \dfrac{-x+8}{2}$

$\longrightarrow y = -\frac{1}{2}x + 4$

$\longrightarrow$ Slope $= -\frac{1}{2}$; y-intercept : $(0, 4)$.

E.x. $3x - 7y = 14$.

Q: Find slope-intercept form. Find slope & find y-intercept

$3x - 7y = 14 \longrightarrow -7y = -3x + 14$

$\longrightarrow y = \dfrac{-3x+14}{-7} \longrightarrow \boxed{y = \frac{3}{7}x - 2}$

Slope $= \frac{3}{7}$; y-intercept : $(0, -2)$.

③ Applications

Key: Interpret slope as rate of change

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$